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HE EFFECT OF THE AERATION OF THE WATER SED IN THE DETERMINATION OF THE MECHANICAL EQUIVALENT OF HEAT

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BSTRACT. In relation to Dr Jessel's experiments two matters are discussed. (i) The ffects of the liberation of dissolved air on the specific heat of water. It is shown that eat of solution, surface energy, and latent heat of vaporization contribute negligible mounts to the specific heat of water. A theory of the equilibrium of spherical air bubbles n fully aerated water is developed. (ii) The reliability of the existing evidence as to the alue of the mechanical equivalent of heat. The conclusion is reached that Dr Jessel's xperiments do not invalidate the work of Jaeger and Steinwehr or of the writers on the nechanical equivalent of heat, and that the value as calculated by Birge, viz. ·1852 × 10^7 erg/cal. at 15° C., is to be accepted.

§ 1. INTRODUCTION

DR R. JESSEL(1) finds experimentally that the specific heat of aerated water is greater than that of de-aerated water. He suggests reasons for this result, but does not discuss the theory exhaustively or quantitatively, and omes to certain conclusions as to the value at present to be accepted for the specific eat of water, i.e., the mechanical equivalent of heat. It is therefore desirable, as guide to experiment, that the theory of the effect of aeration on the specific heat f water should be considered.

§ 2. THEORETICAL CALCULATION OF THE EFFECT OF AERATION ON SPECIFIC HEAT

As the following investigation shows, the calculated effect is very small. Aeration may affect the specific heat of water in three ways. (i) The heat of solution of the air. This has not been determined experimentally, though values have been obtained for some other gases. It can, however, be calculated from solubility data. If N is the mole fraction of the dissolved air, with an external pressure p, Henry's law may be expressed as

$$k\mathrm{N}=p,$$

L and the heat of solution L cal./mole is given by[(2)]

$$\frac{L}{RT^2}=\frac{\partial \log_\epsilon k}{\partial T}.$$

Using the solubility tables of the *International Critical Tables*[(3)], we find that

$$L_{15}=3349 \text{ cal./mole.}$$

This gives for the 0·42 cm³ of air liberated from a litre of water per degree ris 0·063 cal., and a consequent difference through aeration of the specific heat water of the order of 1 in 20,000. This difference is not detectable by any experime hitherto carried out.

(ii) The latent heat of the water vapour required to saturate the air bubbl This will have its maximum effect on the specific heat when all the air liberat from solution by the rise of temperature of the water is retained in the form bubbles in the water. Taking the case of 15° C.:

The volume of dissolved gas liberated per °C. is 0·42 cm³ per litre of water[(3]
The mass of water vapour per cm³ of saturated air at 15° C. is $1{\cdot}27\times10^{-5}$ gm
The latent heat of vaporization at 15° C. is, say, 589 cal./gm.

This gives 0·0031 calories per litre, an increase of specific heat of the order 1 3×10^5.

(iii) The surface energy of the bubbles. The importance of this in its effect the specific heat depends on the size of the bubbles, becoming greater as t average size of the bubbles becomes smaller. It is therefore necessary to obta some evidence as to the relative stability of bubbles of different radii. Dr Jess gives $3a/2T$ for the smallest radius of a stable bubble; this appears to be a m quotation from Callendar[(4)], who gives $\sqrt{(3a/2T)}$ as the greatest radius of a sta
P bubble for the case when P, the vapour pressure within the bubble, is grea
p than p, the external pressure. In the case considered here, in which $P<p$, Callen shows that stable bubbles may have any radius. He is dealing, however, wit bubble containing a fixed mass of air; the following discussion allows for passage of air from the bubble into solution or *vice versa*.

The equilibrium of a spherical bubble in air-saturated water at uniform te perature. Let the external pressure on the free surface of the water be p dyne cr and suppose the water to be saturated with air at this pressure. The vapour press
r, P_t P in a bubble of radius r cm. will be $P_t(1-\text{N})-\frac{2T}{r}\cdot\frac{\sigma}{d}$, where P_t is the saturat
t vapour pressure at temperature t as given in tables, N is the mole fraction of
σ, d, T dissolved air, σ, d are the densities of water vapour and water respectively, an is the surface tension.

p' For mechanical equilibrium, the air pressure p' within the bubble must sati the equation

$$p'+P_t(1-\text{N})-\frac{2T}{r}\cdot\frac{\sigma}{d}=p+dhg+\frac{2T}{r} \qquad \ldots\ldots(1)$$

h where h is the depth of the bubble below the free surface at pressure p.

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* See reference (1), p. 758.

The solubility of the air enclosed in the bubble is increased by the curvature, nd for equilibrium in regard to solution

$$p'=p-\frac{2T}{r}\frac{\rho}{d} \qquad \text{......(2)},$$

here ρ is the air-density at pressure p.

From equations (1) and (2) the radius for equilibrium is given by

$$P_t(1-\text{N})=dhg+\frac{2T}{r}\left(1+\frac{\rho}{d}+\frac{\sigma}{d}\right)$$

nd

$$r=\frac{2T}{P_t(1-\text{N})-dhg}\left(1+\frac{\rho}{d}+\frac{\sigma}{d}\right) \qquad \text{......(3)}.$$

his equilibrium is unstable*, smaller bubbles if in mechanical equilibrium ccording to equation (1) will lose air into the solution, and larger ones will gain air t the expense of the solution. But in either case the rate of change will increase he more r departs from its equilibrium value; so that bubbles of radii near the quilibrium value will predominate. The following table shows values of r in microns iven by equation (3) at depths h below the free surface of fully aerated water.

h (cm.) / t (°C.)	0	0·5	1	2	5
15	87	90	92	98	121
20	63	64	65	69	79
30	34	34	35	36	39
50	11	11	11	11	12
100	1·2	1·2	1·2	1·2	1·2

Effect of surface energy on specific heat. The table above shows that bubbles ormed in aerated water at 15° C. have a radius of about 9×10^{-3} cm. when in equilibrium; bubbles of radii near that value will change only slowly in size, so that we may assume that most of the bubbles present in water that is being heated will have radii of about this value. If the 0·42 cm^3 of air liberated per litre of solution per 1° C. rise remained in the form of n bubbles of this size, the surface energy would be given by $4\pi r^2nT$ while $\frac{4}{3}\pi r^3n=0{\cdot}42$. Substituting $T=74$, $r=9\times10^{-3}$, we obtain for the surface energy $1{\cdot}04\times10^4$ erg or 1 in 4×10^6 of the specific heat. This value may be exceeded through the temporary existence of smaller bubbles, but it leaves a wide margin before any effective contribution to the specific heat could be expected.

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* *Bumping.* The effect of dissolved air in facilitating ebullition is made clear by the above discussion. If water contains sufficient air to saturate it at 100° C., a bubble as small as 1·2 μ will be in equilibrium with the water at 100° C., and water vapour can evaporate steadily into the bubble as the water is further heated, however slow may be the excess supply of heat over that required for losses. On the other hand a bubble of water vapour of this size near the surface of de-aerated water under normal pressure would satisfy the equation

$$P_t=p+2T/r=2p,$$

and consequently it would be in equilibrium with the surrounding water only if the latter were at about 116° C.

It will be seen that none of the effects considered is sufficiently large to accou for the experimental results obtained by Dr Jessel, and that the difference observe by him on aeration of the water requires further investigation. We observed that if a were liberated the rate of flow of the water was irregular, and this disturbed th steadiness in thermal conditions which is essential for accuracy in continuou flow calorimetry. It is to be expected that the electrically heated wire used by D Jessel would be even more disturbed.

§ 3. THE VALUE OF THE MECHANICAL EQUIVALENT OF HEAT

Dr Jessel concluded from his experiment that "if exact and reproducib results for the specific heat of water are to be obtained, air-free water must used" in calorimetric experiments.

This conclusion, if true, means that de-aerated water must be used in a dete mination of the mechanical equivalent of heat. There is no modern* determinati of that constant in which de-aerated water was used, but a consideration of the t available experiments does not support the conclusion that aerated and de-aerat water differ in specific heat.

Dr Jessel does not mention in his discussion the critical reviews which Birge Henning and Jaeger(6) have made of the determinations of the mechanical equivale of heat. Both of these reviews reject all the electrical determinations made previo to 1908. The most valuable of the older determinations is no doubt that of Barn but neither the standard of resistance nor that of e.m.f. which he† used was suf ciently accurately known for the calculation of a value of *J* comparable in accura with that obtainable from the data now available. Birge, after a very thorou analysis of the two determinations, arrives at the conclusion that the electri determination by Jaeger and Steinwehr is in close agreement with the dir determination by the present authors‡. L. H. made 19 determinations of between 15°·62 and 16°·74 C. and 4 at 20°·50 C. Birge points out that in a r duction of our observations which he has made, the mean residual error of t group of 19 observations is 0·0003, while that of the group of 4 is 0·0022. adds that an observation with a residual from 7 to 10 times the mean resid should be rejected according to any of the well-known criteria of rejection, and this ground he rejects the group of 4. He then obtains $J_{15}=4{\cdot}18526 . 10^7$ erg. reduction by Callendar's formula gives 4·18516, in good agreement, and we acc his exclusion of our summer experiments.

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* The view of several authorities on the electrical units is that to be satisfactory a determinat of the electrical equivalent of heat must have been made since 1908, when the London Internatio Electrical Congress was held, for it is not possible to convert into ergs with sufficient accuracy electrical measurements made prior to that date. Thus "modern" is used here to mean since 1

† Barnes's standard of resistance was the British Association ohm, the absolute value of whic uncertain. His standard of e.m.f. was an old type of Clark cell for which in 1902 he gave the va 1·43325; in 1909 he corrected this to 1·4335, while Callendar, on King's experiments, g $1{\cdot}4341 . 10^8$ e.m.u. The resulting uncertainty in *J* is greater than 1 in 1000.

‡ J. S. will be used subsequently for the first of these pairs of authors, and L. H. for the sec

The present authors knew that this group of 4 experiments at 20°·5 C. gave a result for J_{15}, about 1 part in 2000 higher than the other 19 experiments, Callendar's variation of specific heat with temperature being assumed, but as the 4 experiments agreed well together they were not rejected.

Birge gives the following values as a result of his calculations:

J. S. (electrical determination) $J_{15}=4{\cdot}1850\pm0{\cdot}0009\times10^7$ erg/cal.

L. H. (direct determination) $J_{15}=4{\cdot}1853\pm0{\cdot}0008\times10^7$ erg/cal.

Jessel's observations give*

Jessel (de-aerated water) $J_{15}=4{\cdot}1852\times10^7$ erg/cal.

Jessel (ordinary distilled water) $J_{15}=4{\cdot}1905\times10^7$ erg/cal.

He states that his values are correct to 1 part in 2000, that is to 0·0021. It will be seen that his value for de-aerated water agrees with the values found by J. S. and L. H., while his value for aerated water is about 1 part in 1000 greater.

The above figures are evidence that the waters used by J. S. and L. H. have the same specific heat to an accuracy of possibly 1 part in 10,000, and the same as that of de-aerated water to an accuracy of 1 part in 2000. Now J. S. used 50,000 gm. of water in a calorimeter closed to prevent evaporation, and it is probable that this water was not sufficiently saturated with air for the latter to come out of solution when its temperature rose to the region of 15° C. Even if the water were saturated with air the vigorous stirring would prevent the formation of stable small bubbles, the effect of which has been discussed. We used ordinary distilled water under a pressure sufficiently above atmospheric to prevent air bubbles being formed as the water was heated in the 19 experiments at 16° C. The following is quoted from our paper(7) (p. 77) "*Steadiness of conditions.* The most serious temperature variations arise from variations in the rate of flow of the water. At first the flow was regulated by a carefully designed tap placed where the water enters the apparatus. The flow slowly decreased, and was subject to constant small irregularities, the cause of which was not at first evident to us, but later we found these fluctuations in the flow to arise from liberation in the calorimeter of dissolved air. The remedy, moving the tap to the exit side of the calorimeter, was simple enough. The change

* Jessel found the specific heats given in the following table:

J_t in 10^7 erg per calorie at t° C.

Ordinary distilled water			De-aerated water		
Temperature (°C.)	J_t	J_{15}	Temperature (°C.)	J_t	J_{15}
12·3 20·1	4·1944 4·1857	4·1914 4·1897	16·8 22·1	4·1858 4·1780	4·1874 4·1830
	Mean	4·1905		Mean	4·1852

His values have been corrected to 15° by means of an expression of the fourth degree in t calculated by Birge to fit J. S.'s values.

increased the pressure of the water from about 60 to 90 cm. of mercury, and a this higher pressure the water is unsaturated with air until after it has left th apparatus."

A study of the solubility tables will show that the excess pressure of nearl half an atmosphere in the calorimeter would compensate with a wide margin for th tendency of the rise of temperature of about 5° C. to liberate air. It is to be note too that the water passed before entering the calorimeter through a filter of ver fine metal gauze which would remove bubbles larger than the mesh of this gauze

§ 4. CONCLUSION

Neither the theoretical discussion which has been given nor the experimenta results of Jaeger and Steinwehr and of the present authors support the conclusio that aerated and de-aerated water have different specific heats. The value of th mechanical equivalent of heat for both de-aerated and aerated water in which th air remains in solution is $4{\cdot}1852 . 10^{7}$ erg per 15° C. calorie.

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DISCUSSION

Dr R. Jessel. The authors seek to justify their determination of the mechanic equivalent of heat, carried out by the use of water containing a certain amou of dissolved air, by showing that my tentatively suggested theory is not correc This does not, however, alter the experimental fact, viz. that the presence dissolved air increases the specific heat, a fact which has been proved both f crude oils and for petroleum fractions as well as for water. They only show th the theory should be modified to suit the facts.

The ideas expressed in figure 4 of my paper are still probably true. The co tribution of the vapour molecules must be increased by the presence of air; i.e. th formation of steam molecules is greatly facilitated. This increase means that th air in the water in some way unknown helps to break down the highly coaggregate water molecules into simpler coaggregations or even into single molecules. Anoth important point which has not been mentioned is that for aerated water th minimum value occurs at a lower temperature. Barnes and I find it at abo 37° C., Jaeger and Steinwehr at about 33° C. It appears from experiment that th *liberation* of air is not essential. Its presence seems to be all that matters.

In my paper I make no pretence at a standard determination of J_{15}, but to the curacy of my experiment I find that the results agree with the corrected values Barnes, who used de-aerated water, and it is his curve that is plotted. The ggestion made is that his corrected value of 4·1842 should be accepted and not e value 4·1860 or even the corrected value of 4·1853 of Laby and Hercus. Laby d Hercus's results at 20° C., which were self-consistent, seem to have been rown overboard for the simple reason that the deduced value of J_{15} is too high. he presence of air seems an adequate reason for the high result.

I would suggest that the authors settle the matter for themselves by repeating eir experiments, after modifying the apparatus if necessary, at 20° C. with r-free water; and also at higher temperatures, say 30° C. and 40° C. where the fect is much more noticeable, with both air-free water and aerated water. The sults obtained by Bousfield, who also claimed a high degree of accuracy, seem indicate that air has a considerable effect.

It is also suggested that the accuracy of the aerated-water experiments may be some doubt, but this suggestion is entirely unfounded. Practically all the xperiments were made with three or more rates of flow, so that the probable error less than one part in 3000; moreover at temperatures below 35° C. the conditions ere as steady as those for de-aerated water.

Dr H. R. LANG. If the authors examine the results of our experiments on wenty-four samples of oil and petroleum fractions they will surely agree that the ncrease in specific heat caused by dissolved gas is in general too large to be ccounted for entirely in the manner they suggest. I would particularly refer them o the discussion of these results*. Dr Jessel's recent experiments with water onfirm our conclusions. The satisfactory nature of his experiments is proved by he close agreement of his results for de-aerated water with those of Barnes, over he whole range from about 10° C. to 70° C. Using precisely the same method nd apparatus he finds consistently higher values when the water is not de-aerated han when it is, a fact supported by Laby and Hercus's results at 20° C. The uthors' calculations seem to indicate that the air must have some effect additional o those they consider, in order to account for the large difference between the pecific heat of water before and after de-aeration at temperatures above 20° C., ay, where the curves begin to separate; for here the magnitude of the difference s several times too large to be accounted for in the way they suggest. The peculiar hape of the curve found by Lüdin and the Bousfields at higher temperatures also requires explanation, and here again the magnitude of the difference between their results and those of Barnes is much greater, I believe, than can be accounted for by considering only the effects mentioned by the authors.

I hope it will be possible for the authors to carry out some further experiments in the neighbourhood of 40° C. or some higher temperature, in order to test their theory under conditions familiar to them.

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* Lang and Jessel, *J. Inst. Petr. Techn.* **18**, 863 (1932) and Lang, *J. Inst. Petr. Techn.* **17**, 589 (1931)

AUTHORS' reply. As the quantities in question appear to have the same valu to an order of accuracy of 0·1 per cent, it is necessary to define them carefully t prevent the confusion which is present in important writings on the subject. Th is done in the following table.

Energy required to heat 1 *gram of distilled water from* 14·5 *to* 15·5° C.

s_1, s_1'	Water saturated with air, heated at a pressure of 1 atmosphere, part of dissolved air liberated:	s_1 erg	s_1' watt-sec.
s_2, s_2'	Water with air in solution but no air liberated on heating:	s_2 ,,	s_2' ,,
s_3, s_3'	De-aerated water:	s_3 ,,	s_3' ,,

We showed thermodynamically that $s_1=s_3$ to an accuracy of 1 in 10,000, an calculation along the same lines from the solubility data shows that between 15° C and 20° C., s_2 exceeds s_3 by only 1 in 100,000. Using our own measurements an those of Jaeger and Steinwehr, we conclude that $s_1=s_2=s_3=4{\cdot}1852\times 10^7$ in term of the erg.

Dr Jessel measured s_1' and s_3' and found that $s_1'=1{\cdot}001_3 . s_3'$. In his discussio above he rejects our thermodynamic calculations, although he does not point ou any error in them, and only offers in their place qualitative reasoning incompatib with their conclusion. He states that experiment shows that dissolved air increase the specific heat of water and that apparently the liberation of air is not essenti to that increase. We deny that he is entitled to state anything about the specif heat s_2 of water in which air is present but is not liberated, for in all his publishe experiments he used either de-aerated water, or water from which part of the a was liberated as the temperature rose. Assuming he has proved experimentall (though we are unable to reconcile this with our calculation) that $s_1'=1{\cdot}001_3 . s$ that does not prove that s_2 is not equal to s_3 to an accuracy of 1 in 10,000, which an important issue.

Dr Jessel reasserts that Barnes's absolute value of J is to be accepted, and ignor the criticism, made by every authority on electrical units who has discussed Barnes experiment recently, that his electrical standards of resistance and e.m.f. are t much in doubt to make his result of value. We believe Jaeger and Steinwehr value, as an *absolute* determination, to have about ten times the accuracy of Barnes'

Dr Lang discusses whether $s_1'>s_3'$. He does not accept our thermodynamic calculation which gives $s_1'=s_3'$ to an accuracy higher than 1 in 10,000. He explai our disagreement with Dr Jessel's experimental result $s_1'=1{\cdot}001_3$ by saying w must have left some effect out of account, but makes no suggestion as to wh effect has been omitted. He considers the facts that (i) he and Dr Jessel found greater than s_3' for twenty-four oils, and (ii) Jessel's value of s_3' agrees close with Barnes's value over the temperature range 10° to 70° C. to afford addition evidence that $s_1'>s_3'$ for water. Both these facts and our thermodynamic calc lations can be reconciled if it is assumed that the liberation of air in Callendar an Barnes's calorimeter changes the temperature-distribution as compared with th distribution which obtains when no air is liberated, and that this leads to values s_1' which are too high by about 1 in 1000.

Preliminary experiments in our determination of J showed that Callendar's elementary theory, which Jessel uses to reduce his observations, is very difficult to realize in practice. In order to avoid this difficulty, we abandoned Callendar's method of eliminating heat-loss and devised a new method in which the loss was 0·04 per cent, and could be determined certainly to ± 10 per cent of its value. In Jessel's experiments different calorimeters were used with the de-aerated and the aerated water. The heat-loss for the one used with de-aerated water was apparently 1 or 2 per cent, and for the other was higher. Our experience with this type of calorimetry makes us believe that an error of 1 in 1000 in the values of s_1' and s_3' might arise under such conditions.

The magnitude of the effect under discussion can also be calculated from the heat of solution as obtained from solubility data. The specific heats of air-free and aerated water may be compared by considering a cycle in which water is saturated with air at 20° C., the water is cooled to 15° C., the air is drawn from solution at that temperature, and air and air-free water are heated separately to 20° C., completing the cycle. The solubility data used above give $L_{20} = 3120$ cal./mol., with a consequent difference of specific heats over the range 15 to 20° C. of 6×10^{-6} cal./gm. So, allowing for uncertainties in the solubility data, we may say that the difference due to this cause is not more than 1 in 100,000.

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THE COLOUR-MATCHING OF TUNGSTEN-FILAMENT LAMPS BY MEANS OF A SINGLE PHOTO-CELL AND COLOUR FILTERS

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ABSTRACT. A method for colour-matching tungsten light-sources is described, i which one photocell and two coloured filters are employed. The ratio of the total trans missions of the two filters, as measured by the photocell, is used as an empirical measu of the colour temperature of the source, a tungsten-filament lamp. For certain lamps an photocell-filter systems the relation between this ratio and the lamp-voltage is approx mately linear. An example of the use of the method in checking the calibration of standar of colour temperature is described, reference being made to a set of N.P.L. colou standards.

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§ 1. INTRODUCTION

A METHOD of using two photocells of different colour characteristics for th colour-matching of tungsten-filament electric lamps has been described b G. T. Winch(1).

Its basis is as follows. The ratio between the energy radiated by the lamp filament in one spectral region to that radiated in another spectral region depen on the colour temperature of the source. The spectral regions having been define the ratio may be used as an empirical measure of the colour temperature of th source. Two such regions are selected by the two photocells, and the colour tem perature of the source then determines the ratio of the outputs of the two cells. F colour-matching in practice, the more sensitive cell is shuttered to give unit rat when the cells are illuminated by the colour standard. The latter is then replac by the test lamp, whose voltage is regulated until unit ratio is again indicated. E the use of a null method it is secured that attainment of unit ratio shall correspo to zero deflection of a sensitive galvanometer, and thus a high sensitivity is attaine This method entails the use of two photocells, which increases the cost of th apparatus above that of the simple photometer, and involves the use of a lar diffusing window in the case of directional measurements, with some uncertain as to the perfection of the diffusion unless the light-sources are of similar size an design.

With these disadvantages in view, and also as a matter of theoretical intere another method, in which the same principle is applied in a different way, described in this paper. It has the advantage that an ordinary photoelectric phot meter, with one photocell, can be used with the simplest of additions.

§ 2. THE METHOD

The present method employs one photocell, of sufficiently extended colour-nsitivity, with two interchangeable colour filters to select the two regions of the pectrum. The ratio of the cell-outputs with the two filters successively in position an be used as a measure of the colour temperature of the light-source. In this case vo separate readings are taken to determine the ratio, and a null method cannot be sed; the procedure for colour-matching described below is adopted.

This method is a particular case of a general method. If between a light-source nd a receiver—e.g. an eye or photoelectric cell—various coloured filters are uccessively interposed, the corresponding outputs of the receiver, together with nowledge of the properties of the filters, give information concerning the relations xisting between the spectral distribution of the energy radiated by the source and the pectral sensitivity of the receiver. The amount of information thus obtainable depends upon the number and properties of the filters, and the data available concerning hem. In particular if either the quality of the radiation or the properties of the eceiver remain unchanged, variation in the factor that is subject to change can e deduced from the readings on the set of filters. For instance, if a source of constant colour be used, the colour characteristics of various human observers can e classified according to the ratio (the "Y/B ratio") which each obtains for the ransmissions of two standard yellow and blue filters; or the performance of a photoelectric cell may be assessed by comparing the overall transmissions of a set of colour filters as measured with the cell with those determined on the visual basis, all he values being related to a source having a constant colour temperature (2). Alternatively if a receiver of constant properties, e.g. a photocell, is employed, the ransmission readings for a set of filters will afford information as to the colour of he source. In the case of a tungsten-filament source, for which the energy-distribution bears a unique relation to the colour temperature, the transmission of a single filter will, in general, bear a unique relation to that colour temperature, and may be used as a measure of it. In certain cases the relation will not be single-valued; and if the filter be strictly neutral, its transmission will of course be invariable.

In the method herein described, however, two filters are employed for the practical reasons that a higher rate of change of the measured quantity with colour temperature of the source is obtained for the usual cells and filters available, and that if the filters are suitably chosen the photoelectric photometer may always be used at nearly its full scale-reading, with advantage in regard to precision.

The procedure for colour-matching two lamps is as follows. A photoelectric photometer is set up with the cell exposed to light from the colour standard, which is adjusted to the desired temperature. Two readings of the photometer are taken with, say, a red glass and a green glass in front of the cell. The green/red ratio is thus found. The colour standard is then replaced by the test lamp which is to be matched with it, and the procedure is repeated for two or more voltages on the test lamp. Appropriate interpolation gives the voltage on the test lamp required to give the same green/red ratio as the standard, i.e. to colour-match it.

§ 3. PRACTICAL CONSIDERATIONS

The results quoted below were obtained with an emission photocell in the we known thermionic-valve bridge circuit. The photometer need not be describe here, but the following considerations assume the use of such a photomete although many of them are of general application.

The photocell. The photocell should have a wide spectral sensitivity band ar stable characteristics. A rubidium-on-silver thin-film vacuum cell with centr plate cathode is a suitable type. The cell may have an illumination-respon characteristic of one of three types, namely (i) strictly linear; (ii) not linear, but similar form for all wave-lengths and such that the cell will always give the san measured value for the transmission of a filter, no matter what illumination-valu of given colour temperature is used; (iii) not linear, and of different form for lig of different wave-lengths.

In cases (i) and (ii) the procedure outlined above is valid. In case (iii), howeve the cell will exhibit the Purkyňe effect, and the green red ratio for a given colo temperature will vary according to the degree of illumination employed. It w then be necessary to work always with the same reading for one or other filter. Th is best done by having the light-source movable on a photometer bench and alwa adjusting its distance from the diffusing cell window so that a definite consta reading is given by the photometer for one of the two filters. The reading on th other filter will then have one definite value for a definite colour temperature, an so will the ratio. In practice it is useful to set the reading for the less dense filter the maximum scale-reading of the photometer.

A diffusing window in front of the photocell is a necessity, and is of particul importance in the case where the light-source is movable.

The filters. Filters suitable for use with the rubidium cell are a selenium red an a signal green glass. These should be optically finished, of good uniformity, an carried in a mechanical slide to ensure accurate placing and replacing in the pa of the light. It may be necessary to combine the selenium glass with a neutral filt to give it an effective density more nearly equal to that of the green glass, so th both readings shall fall near the upper end of the photometer-scale, with conseque improvement in precision.

The main source of error likely to be encountered in respect of the filters change in temperature, with a corresponding change in transmission. To keep th error small, a change of temperature exceeding 1° C. should not be tolerated.

§ 4. RESULTS

The following description of results obtained by the above method will illustra its application.

(i) The method was used to investigate the mutual agreement, at a nomin colour temperature of 2360° K., of a set of six colour standards, and to readju individual lamps to the mean of the group.

The lamps used were N.P.L. working standards 87G-L vacuum squirrel-cage lamps of nominal rating 100 V., 60 W. and had been calibrated visually. The set-up was as described above with a rubidium cell showing Purkyňe effect. The measured transmission of the red filter was reduced by cementing it to a neutral filter. The photometer was mounted on a bench and fitted with a diffusing window.

Lamp 87G was selected for use as a reference lamp against which to check the other five. Its stated voltage for the colour temperature of 2360° K. was 85·9.

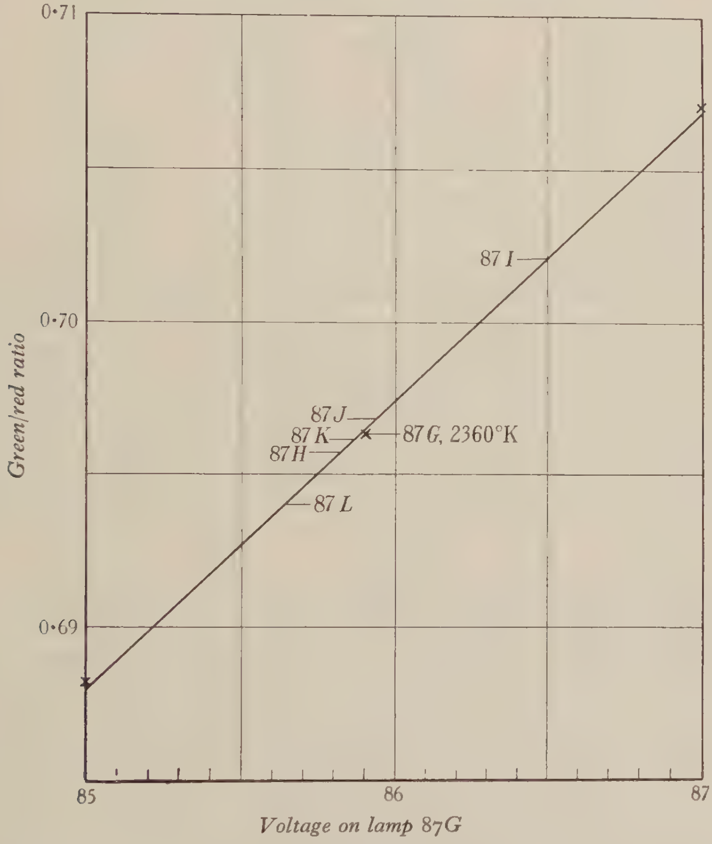

Figure 1.

Its green/red ratio was measured at 85, 85·9 and 87 volts, the red reading being the greater throughout and so being set at a constant (full-scale) value. The relation between voltage and green/red ratio, figure 1, is seen to be a straight line over this range.

The green/red ratio was then measured for each of the other five lamps, run respectively at the voltages stated in the calibration charts to give a colour temperature of 2360° K.; see the table, column A. If the stated voltages had been exactly the correct ones these five ratios should have agreed precisely with that for 87G at 85·9 volts. This agreement did not occur, the observed ratios being as in the table, column B.

Table.

Lamp	A Nominal voltage for 2360° K.	B G R ratio at voltage in column A	C Voltage on 87G to give ratios in B	D* Deviation from mean of column C	E Corrected voltages for colour temperature of 2360° K.
87G	85·9	0·6963	85·9	0	85·9
87H	85·5	0·6957	85·8	−0·1	85·6
87I	85·6	0·7020	86·5	+0·6	85·0
87J	85·3	0·6968	85·9	0	85·3
87K	85·3	0·6961	85·9	0	85·3
87L	85·2	0·6940	85·6	−0·3	85·5
			Mean 85·9		

* A deviation of 0·1 volt corresponds to a deviation in colour temperature of 1° K.

If now we pick off from the graph for 87G, figure 1, the points correspondi to the ratios in column B, we obtain as abscissae the voltages at which 87G wou colour-match the various lamps at their nominal settings for 2360° K., column The spread in these voltages represents the spread in the original calibration of t group, plus divergences due to subsequent use. The mean of these voltages, giv at the foot of column C, is the voltage at which 87G operates at a colour temperatu of 2360° K. *as based on the present mean calibration of the group of six lamps.* Th voltage happens to be 85·9, the stated value for 87G. This agreement is fortuito and arose from the fact that 87G happened to represent exactly the mean of t group.

Column D of the table gives the deviations of these voltages from the mea i.e. the departure, in terms of the voltage applied to 87G, of the colour of each lan from the mean of the group at 2360° K. Now since all the lamps were of the sa type, operating at very nearly the same voltage, a small variation in the voltage 87G will correspond almost exactly, as regards range of colour temperature, wi an equal voltage-variation about the same fixed value on any other lamp in t group. We can thus readjust each lamp to the mean of the group. For instan 87I at a nominal colour temperature of 2360° K. gave a high green red rati figure 1, indicating that the stated voltage was too high by an amount equivale to an increase of 0·6 volts on 87G. The stated voltage of 87I should therefore reduced by a like amount. By such a process we arrive at the values in column which are the voltages at which all six lamps accurately match in colour, and at whi they represent the mean 2360° K. point for the group, *on the basis of the prese calibration.*

A second complete redetermination of the values in column E led to the sa figures within 0·1 volt in every case, showing that the precision of the meth corresponds to a colour-temperature range of about ± 1° K. at 2360° K.

(ii) The form of the relation between green red ratio and lamp voltage w found in case (i) to be linear over a range of 2 volts for 87G. As a matter of intere his range was extended to 40 volts, and the measurements of the ratio were made t intervals of 5 volts with two entirely different cell-filter systems. The results are hown in figure 2. Curve A was obtained with a rubidium cell which showed the urkyňe effect. The filters were a selenium red glass, attached to a neutral filter, and signal green glass. The conditions were not precisely as in case (i).

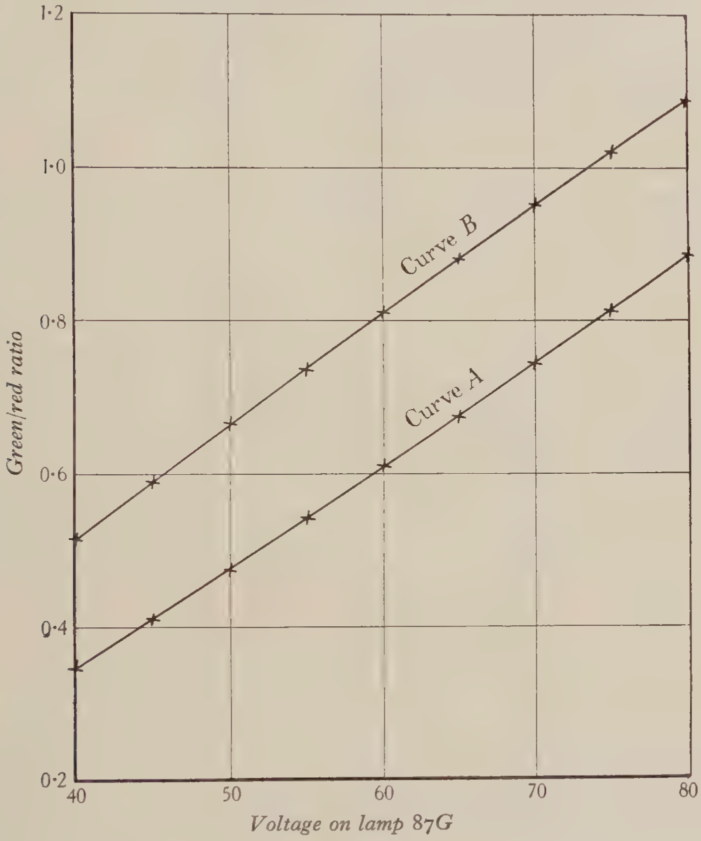

Figure 2.

Curve B was obtained with a thin-film potassium-on-silver cell. The filters in his case were a dark-orange-plus-orthogreen combination, and a signal green glass. The relations are not precisely linear, but so nearly so that linear interpolation over ranges of a few volts does not involve errors as large as the experimental errors. In olour-matching a pair of lamps, therefore, the green/red ratio for the standard having been determined, measurement of the ratio for two voltages only on the test lamp gives sufficient data to enable the colour-match voltage to be found by interpolation, provided it is close to the measured points.

§ 5. CONCLUSIONS

The photoelectric determination of the ratio of the transmissions of tw coloured filters gives a convenient and precise method for colour-matching tun sten-filament electric lamps. The method has been successfully used to ascerta the spread in the performance of a group of colour standards of the vacuum 100-W 60-W. squirrel-cage type, and to readjust them to the mean of the group.

For lamps of this type the relation between the transmission ratio and the lam voltage was found to be practically linear over large ranges, for two differe photocell-filter systems.

The results of the experimental work show that the precision and reliability the method are such as to justify consideration of its extension to the establishme of a scale of colour temperature on the basis of a set of calibration points ind pendently derived.

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SPECTROSCOPIC OBSERVATION OF RECURRENT PHENOMENA IN DISCHARGE TUBES

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ABSTRACT. A method for the synchronous photography of irregular moving striations is described. It consists in forming an image of the positive column of a discharge along the slit of a spectrograph and covering and uncovering the latter with a mechanical shutter made to synchronize with the striations by a photoelectric device. The spectra obtained show variations in intensity along each line, corresponding to the intensity of the line in the bright and dark phases. Direct photography also of the actual striations is described.

The photographs obtained for pure argon show that there is no appreciable recombination in the dark phase, while those for argon-mercury mixtures show excitation of only the mercury lines in the dark phase. These results have been explained by assuming that in the dark phase the electron energies are high enough to excite the mercury lines and are therefore too high to permit of recombination taking place. It is only in the bright phase that the energies are large enough to excite the argon lines. The optical results described are in accord with data obtained by Pupp by an electrical method. Their bearing on the work of Ladenburg is also discussed.

§ 1. INTRODUCTION

Stroboscopic observations of regularly occurring phenomena are usually made with a synchronous slotted rotating disc. The aim of the present investigation has been to devise a method applicable to phenomena which do not occur with constant frequency and to which, therefore, the ordinary method is not applicable. This has been achieved by arranging to have a mechanical shutter opened and shut in synchronism with, and directly controlled by, the individual events. Although primarily designed for an investigation of moving striations in low-pressure discharges, the method is general and could have wider applications than those indicated here.

Briefly, the method consists in applying (for example by means of a photocell) voltage impulses caused by the individual events to a thermionic valve amplifier which feeds an output stage capable of giving a large current; this current is passed through a modified form of Einthoven string galvanometer, so that when the fibre is deflected it uncovers a slit through which direct observations of the events are made.

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§ 2. CIRCUIT

The circuit is shown in detail in figure 1. Light from the discharge tube T, i which moving striations were present, was admitted to a vacuum photocell through a fine slit in an earthed metal box. The slit could be either vertical o horizontal and its dimensions could be varied to control the intensity of the ligh incident on the photocell. The current through the photocell developed a potentia difference across a resistance R_1 in such a way that each striation, as it passed th photocell, gave rise to a negative voltage impulse which was applied to the contro grid of a screen-grid valve V_1 with variable amplification factor.* The output fro

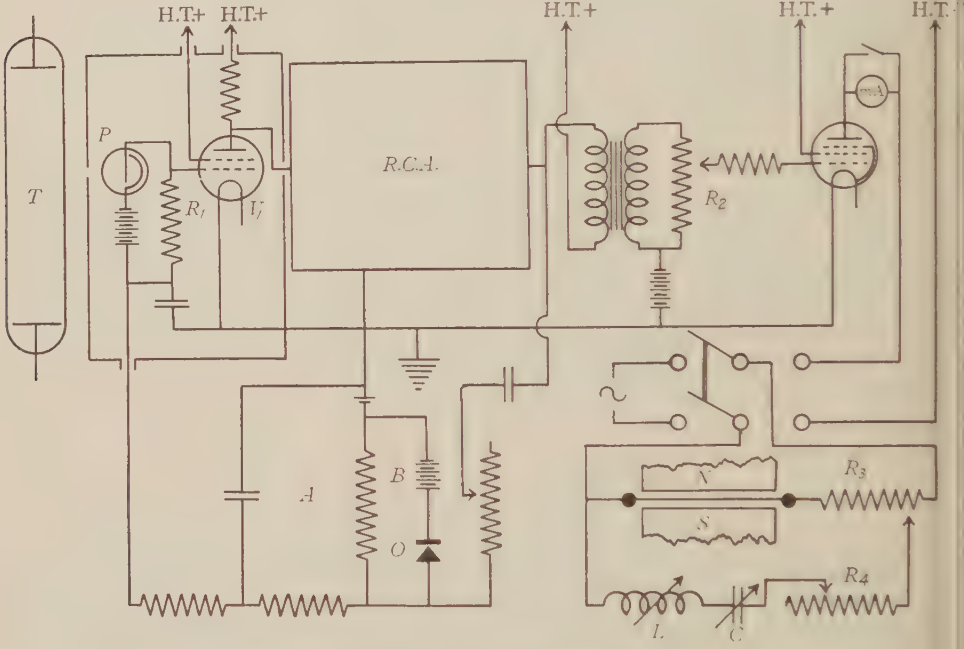

Figure 1. Circuit diagram.

the latter was passed into a three valve resistance-capacity coupled amplifier RC of conventional design, which in turn fed an amplifier, consisting of two pentode in parallel, through a transformer and a potentiometer device R_2 which regulat the input to the pentodes. The anode current of the pentodes operated the electr magnetic shutter.

Since it was desirable to have the current in the shutter unidirectional, the tw pentodes were used with a large steady negative voltage applied to their contr grids so that the anode current could only flow during the positive half-cycle of th input voltage from the previous amplifier. When the shutter was being operate the average anode current to these valves, measured on a moving-coil milliamet mA, was adjusted to be between 70 and 90 mA., varying in extreme cases by no more than 5 per cent, while the quiescent current never exceeded 3 mA.

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* Osram VS 24.

† Osram PT 16.

It was often necessary when photographic records were being made to leave the apparatus working continuously for several hours, and during this time the intensity of the light from the discharge tube was liable to vary considerably. The special function of the valve with variable amplification factor was to keep the output current constant in spite of such changes. This was accomplished by having its control grid connected to the output of the three-valve amplifier by a rectifying circuit A. This well-known circuit was so adjusted that when the output from the amplifier exceeded a certain voltage determined by a battery B, the control grid voltage of V_1 began automatically to be made more negative by the voltage developed across a copper oxide rectifier O, and hence the amplification produced was decreased. This system was more flexible than any utilizing the rectifying properties of the pentodes.

The pentode output current was adjusted initially to the required value by varying the input to the pentodes by a potentiometer device R_2, which while it controlled the amplitude had a negligible effect on the phase.

§ 3. CONSTRUCTION AND OPERATION OF SHUTTER

The design of the magnet and shutter is shown in figure 2; it applies the principle employed in the Einthoven string galvanometer. The pole pieces were bored as shown and were mounted at the centre of two end plates which were held together by the bobbin cores. The two field coils were wound for operation on the 220-volt d.-c. mains and were cooled by passing water through two spirals of copper tubing which formed the ends of the former of the coil, as shown. The inner ends of these spirals were soldered into a brass cylinder which was spaced from the iron core by a watertight brass ring at either end, permitting water to flow from one spiral to the other. Efficient water cooling was necessary to ensure constant temperature of the fibre mechanism during operation. As a further precaution, a small shield of copper foil was placed over each winding and soldered to the copper spirals on the side of the coil adjacent to the fibre.

The width and horizontal position of the spectrograph slit could be varied by means of thumb-screws A and B; these controlled two long levers, not shown, which in turn operated the slit. The fibre was stretched over two ebonite bridges C mounted on a brass carrier which could be withdrawn by slackening two thumb-screws D. Connexions were taken from the ends of the fibre to the terminals E. When the thumb-screw F was slackened, the rod G could slide freely in the ebonite terminal block, allowing the tension of the fibre to be adjusted as described in § 5.

The fibre could be set accurately parallel to the jaws of the slit by rotating its holder about a pivot K by means of the screws L. Horizontal movement of the fibre holder was effected by means of bevel gears H, which in turn were controlled from the back of the spectrograph by a long rod with a universal coupling.

Since oil damping of the fibre would have been impracticable, the motion was

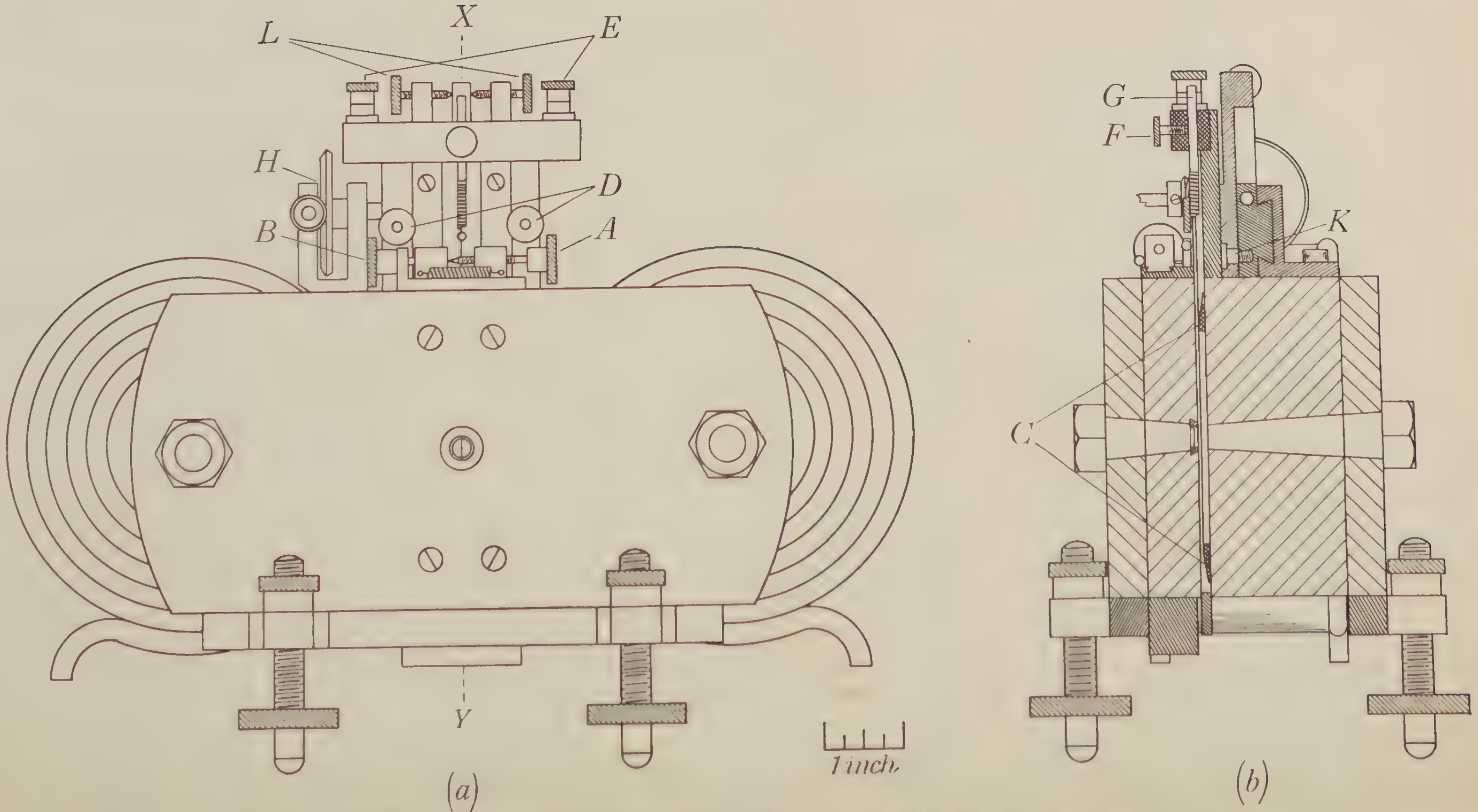

Figure 2. (*a*) Elevation of electromagnetic shutter. (*b*) Sectional elevation through *XY*.

damped electrically by an Irwin resonant shunt(1). In this method the fibre is shunted by a series tuned circuit LC, figure 1, with a frequency closely equal to that of the fundamental mode of the fibre. By means of the auxiliary resistances R_3, R_4, the amount of damping could be varied within wide limits. This system worked admirably.

§ 4. CONSTRUCTION OF DISCHARGE TUBES

Tube 1, figure 3, was used for discharges through argon-mercury mixtures and had nickel electrodes in the form of hollow cylinders, mounted on valve pinches. It was permanently fused to the pumping train and was baked for several hours under high vacuum before use, after which the electrodes were degassed with an eddy-current heater. Since only very small traces of mercury vapour were required, the tube was isolated from the mercury diffusion pump by a liquid air trap. Mercury vapour was admitted from a side-tube, which contained a drop of distilled mercury, by opening a tap for a few minutes at regular intervals. The tap

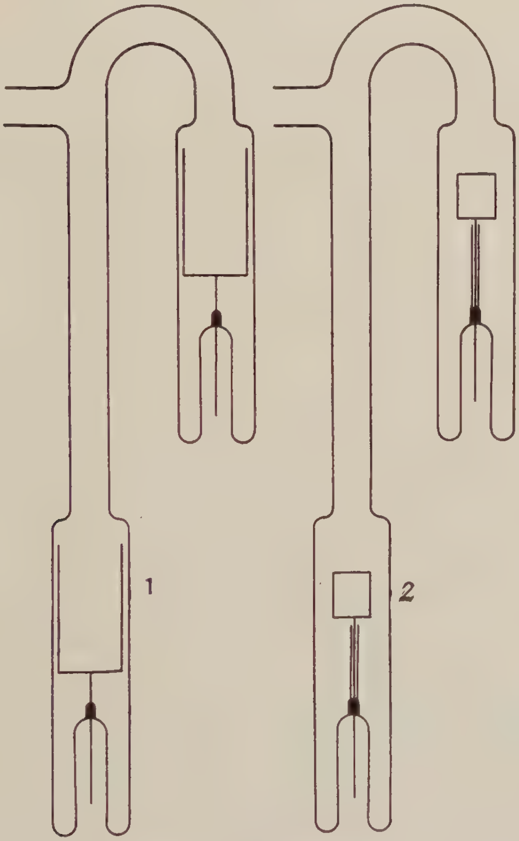

Figure 3. Discharge tubes.

was lubricated with apiezon grease, and apart from mercury lines the spectrum was entirely that of argon.

Tube 2 was intended for work on pure argon and on the auroral line of oxygen. It was similar in shape to tube 1, but had thick degassed tungsten plates for electrodes. Before the admission of argon the gas was first purified by means of a misch-metal arc, and in tube 2 a final treatment was given in the tube itself by heavy sputtering of the tungsten.

Tube 3 was a straight commercial tube, 50 cm. in length and 1·5 cm. in bore. It contained neon, and had hollow electrodes of sand-blasted steel in short T-pieces at the ends.

§ 5. APPLICATIONS

Spectra of moving striations. The magnet described in § 3 was designed to rest on an extension of the base of a glass-prism spectrograph, so that the electromagnet slit was almost in the position of the original spectrograph slit. By means of thumb-screws the position of the electromagnet was then adjusted so that the slit was accurately at the focus of the collimator lens. V-shaped stops into which the levelling-screws of the electromagnet fitted were provided, allowing the electromagnet to be removed when required and replaced accurately in position.

For spectrographic work, there are two possible positions for the discharge tube. (i) The tube may be connected horizontally to the pumping train and an image of it focused on the slit so as to cross it at right angles. By suitably adjusting the phase-shift in the amplifier with a phase-shifter it would then be possible to obtain spectra of either the bright or the dark periods of the moving striations. This method was not used. (ii) When vertical, the tube may be focused longitudinally on the slit. Spectrograms obtained in this way show regular variation in intensity along each line, corresponding to the varying intensity of the line in the bright and dark periods. A typical example for neon is shown in figure 4; the broad lines are due to the use of a wide slit. This method has the advantage that the spectra of both periods can be photographed simultaneously and direct comparison of the intensities can be made on the same plate. It has therefore been used exclusively.

Before each exposure, the fibre was given a known tension by inverting the holder and suspending weights from it. The loaded fibre was then replaced in the shutter, the water flow was started and the field current was switched on. After allowance of about 30 minutes for steady-temperature conditions to be attained, the fibre was allowed to resonate at its natural frequency, a separate valve oscillator being used as the source of current. With the phosphor-bronze strip, 6·3 cm. long by 0·17 mm. wide, under a tension of 70 gm. weight, used for most of the work, the frequency was about 1700 sec.$^{-1}$ The Irwin shunt was then connected and the inductance was varied until the vibration of the fibre, as seen in a microscope, was completely suppressed. After this, the fibre was oriented until it was accurately parallel to the jaws of the slit. The width of the slit was always adjusted with the magnet in position by observing the spectrum of the discharge in the back of the pectrograph. Then, with only the quiescent pentode current flowing, the fibre as moved over so as just to cover the slit. Finally the photocell was placed close o the discharge and exposed to it, when the striations could be seen in the back of he spectrograph in the various monochromatic wave-lengths emitted by the tube.

Direct photography of the striations. The apparatus was also used to obtain photographs of moving striations in the positive column of a discharge in neon. Tube 3 was first of all focused on the ground-glass screen of a camera. After the adjustment described in the previous paragraph, the shutter was interposed between the camera and the tube, so that only light which had passed through the shutter from the tube could reach the camera. Figure 5 was obtained in this manner with n exposure of 20 minutes on a fast panchromatic plate, and represents about cm. of the tube. A photograph of this length is obtainable only when the velocity of the striations is constant throughout the region photographed. The dark line at X is due to a small appendix on the discharge tube.

Visual observations of the positive column made with tube 2 in pure argon showed that although the contours of the individual striations were very similar to those of figure 5, they were more widely spaced from each other. It should be noticed that the striations are actually less elongated than appears from such photographs, since they move during the time that the shutter is open. The fractional distortion depends on the conditions of adjustment. Its chief drawback is that it prevents us from observing accurately how the spectrum varies just at the edge of the bright part of the striation. The spectra of the main bright and dark periods can be recorded faithfully.

§ 6. DISCUSSION OF OBSERVATIONS

By means of the procedure described in § 5, spectrograms were obtained with tube 2 containing pure argon at pressures of 2–3 mm. The tube was run at 740 V. with currents of 40–80 mA. A careful examination was made of the spectra of the dark periods but there was no visible trace of high series lines or series continua, such as might be expected if recombination took place in that region.

It has been shown by Kenty(2) that recombination of electrons with positive ions of argon in the gas phase is negligible when the electron temperatures exceed about 0·5 e-V. Hence we may conclude that the electron temperatures in the dark phase exceed this value in our tubes. This conclusion is in accord with the work of Pupp(3), who by an oscillographic method has made probe measurements in moving striations in the positive columns of discharges through argon. Since the current-densities were of the same order,* in this work and in our own, we are probably justified in assuming that Pupp's results can be applied with fair accuracy to our discharges in pure argon. The results obtained in his tubes show that the minimum value of the electron temperature in any phase of the striation is not less than 1·5 V. Energies of this value preclude any possibility of appreciable recombination. Pupp's work also shows that the values obtained for ion-concentration repeat themselves very closely through several striations. Since the ions do not recombi in the gas, it follows that the average rate at which they are generated at any pa of the positive column is equal to the rate at which they are lost from it (ultimatel to the walls) under the action of the electric fields and concentration-gradients.

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* The range of Pupp's current densities was 0·05–0·10 A.cm.$^{-2}$, while ours was 0·03–0·06 A.cm.$^{-2}$

Curve (i), figure 6, gives the number of ions generated sec.$^{-1}$ cm.$^{-1}$ along the ax of the positive column. It was obtained by substituting the numerical values fo electron-concentration and temperature as given by Pupp,* in the kinetic-theor expression of Killian$^{(4)}$ for the rate of ionization by collision. In order to calcula the rate of diffusion to the walls, it would be necessary to consider an expression the form$^{(5)}$

$$D\left[\frac{e}{kT}\frac{dV}{dr}+\frac{1}{n}\frac{dr}{dr}\right],$$

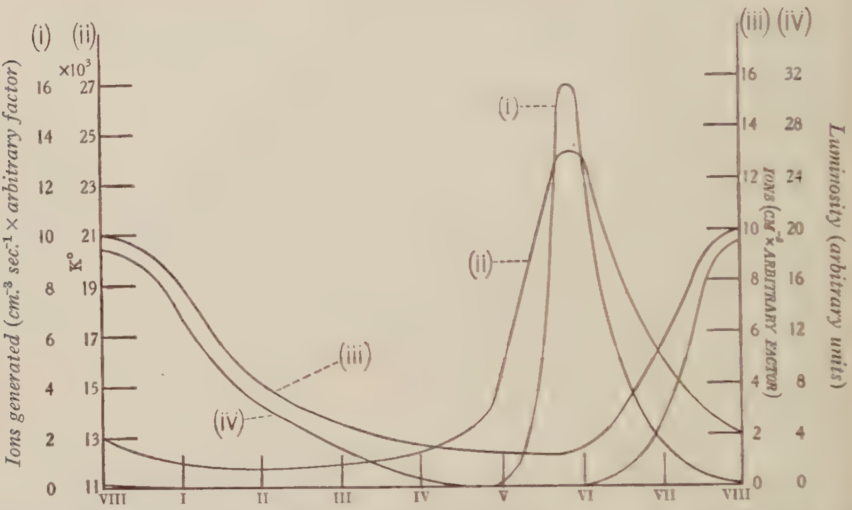

Figure 6. Curves illustrating the variation through a striation of (i) the rate of generation of positi ions; (ii) the electron temperature; (iii) the concentration; (iv) the light intensity. The absciss scale is identical with that used by Pupp.

where D is the diffusion coefficient of the ions. Now the value of D is extreme sensitive to such factors as purity of the gas and temperature$^{(6)}$, and the conditio are unfortunately not well enough known to enable a reliable value of D to estimated. Thus it is not at present feasible to make an exact estimate of the ra of diffusion of ions to the walls, but it is certain, from the optical indications, th were such a calculation possible it would be found that both the ionization an diffusion curves, when integrated over a complete striation, would give the sam value. In other words just as many ions pass to the walls as are generated.

Figure 7 was obtained with tube 1 filled with argon at a pressure of 2 mm., an just sufficient mercury vapour to give the lines of both elements with comparab intensities—probably less than 0·03 per cent. This photograph, again obtained described in § 5, shows the spectrum of a dark period along the centre and that a bright striation on either side. The outstanding feature is the persistence of t mercury lines λ 5461 and λ 4358 through all phases, although argon lines of equal and greater intensity in the bright phases are relatively very weak in the dark phase(7).

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* We assume, as usual, that the electrons have a full Maxwellian distribution.

The question arises as to the cause of this selective excitation. In the dark phase the excitation of the mercury lines could be due either to the persistence of excited systems from the bright period, or to direct excitation by direct electron collision in the dark period. If persistent excited systems were to be important, they would have to possess a lifetime of more than about $5 \cdot 10^{-4}$ sec., since the frequency of the striations is of the order of 10^3 sec.$^{-1}$ The persistence of ordinary excited states for this time is obviously out of the question, and even if either of the metastable argon atoms, with excitation potentials of 11·50 and 11·67 V., were present in the dark period, the probability of their exciting the mercury atom to the 7^3S_1 state, with an excitation potential of 7·6 V. by a collision of the second kind, with consequent emission of λ 5461 and λ 4358, would be negligible owing to the large energy-difference involved. There is a slight possibility that metastable mercury atoms, if present, could be excited to the 7^3S_1 state by electron impact, since Pupp's data for electron temperatures show that energetically this is possible.* However, apart from energy considerations, the concentration of these metastable atoms is hardly likely to be sufficient to account for the intensities of the observed lines, since a cumulative process is involved. The main excitation process then appears to be by direct collision of electrons with neutral mercury atoms.†

Curves (ii), (iii) and (iv) in figure 6 were derived from diagrams given by Pupp, and represent the variation of electron temperature, concentration and light-intensity through the striations.‡ They show that the region of maximum electron temperature precedes the front boundary of the bright striations, a result which appears at present to be inexplicable. We are therefore forced to the conclusion that either this result is spurious, or some fundamental fact bearing on the mechanism of moving striations still remains to be discovered. However, Pupp's results definitely show that throughout a cycle sufficient electrons are present with energies large enough to excite the mercury lines observed, although it is only in certain regions that strong excitation of the argon is possible.

It must be emphasized that precautions must be taken in work on intensities in spectra to ensure that the positive column shall be uniform, although this has not always been done. If the positive column is not uniform, great care is needed in interpreting the results obtained, for the presence of moving striations is liable to lead to large errors. For instance, the fact that large concentrations of atoms in many of the excited states are present only in the bright region indicates that in transverse absorption experiments, without synchronization of the source of light with the striations, one cannot hope to get more than about 50 per cent absorption when these states are involved. It is only when metastable atoms with fairly long lifetimes are responsible for the absorption that it will be appreciable in all phases, since the long lifetimes of these states will tend to maintain a reasonably high con-

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[*] Since in the Maxwellian distribution there will be an appreciable number of electrons with energies twice that of the mean value.

[†] Although Pupp's data refer to discharges through pure argon, we can probably apply them to our case without appreciable error.

[‡] The curves shown refer to the axis of the tube.

centration through the striation. When the emission spectrum of a discharg showing moving striations of this type is examined laterally without any strob scopic device, it is also obvious that the spectrum obtained may show an anomalou distribution of intensities among the observed lines due to the preferential emissio in the dark period of some lines of the type which we have found. Finally, in th case of measurements of longitudinal absorption, as in the work of Ladenburg(8 the same kind of averaging effect is present. It is only to be expected, in accordan with his results, that electron temperatures deduced from data obtained in this wa will not give anything better than order-of-magnitude agreement with resul obtained by electrical methods, such as those of Seeliger and Hirchert(9), in whic the probe is used continuously, and not intermittently in synchronism with th striations if such are present. Used intermittently, the probe will give true tem peratures for the particular phase of the striation during which it is in circuit(2, but used continuously it will merely give an average temperature which is n necessarily the same average as that yielded by the optical method.

§7. CONCLUSION

It is possible that the circuits employed here could be improved in detai those used were found quite satisfactory for this investigation and permitted th use of apparatus already available. With modern high-efficiency valves, adequa amplification could be obtained with fewer stages. Further development coul apparently be effected by application of the method to the observation of plasm oscillations(10), by modifying the circuits and using a Kerr cell or other shutt suitable for very high frequencies.

The results for discharges through argon-oxygen mixtures will be discussed a forthcoming publication.

§8. ACKNOWLEDGMENTS

We wish to thank Prof. Emeléus for suggesting the problem and providi facilities for the experimental work, and for criticism and advice throughout Our thanks are also due to Mr J. Wylie, B.A., for valuable help with the desi of the magnet which was constructed in the Physics Workshop by Mr A. Buic The commercial neon tube was kindly lent by the General Electric Compa through Mr Morrison, of the Belfast branch.

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THE MAGNETIC MOMENT OF THE MANGANIC ION

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STRACT. The magnetic susceptibility of manganic acetylacetone has been measured a powdered specimen from 292° to 16·9° K. The law $\chi\,(T+5\cdot5)=$ a constant is obeyed vn to about 75° K., below which the susceptibility increases somewhat more rapidly n is indicated by this law. The magneton number for the Mn^{+++} ion, for which no reliable a were previously available, is 4·98 as calculated from the above relation. This is in d agreement with the requirements of the Bose-Stoner theory.

§ 1. INTRODUCTION

THE available data on the magnetic properties of substances containing the manganic ion Mn^{+++} are very meagre on account of the fact that these compounds are generally very unstable. They are practically restricted to Weber's(1) asurements on solutions of certain manganic salts and Honda and Soné's(2) on nganic oxide Mn_2O_3. The former measurements were made at room-temperature ly and, on the assumption of the validity of Curie's law, the most reliable data, ose on manganic ortho-phosphate, $MnPO_4$, lead to a magneton number of 5·03. the other hand, when Honda and Soné's values of the magnetic susceptibility re plotted on the customary $\{1/\chi, T\}$ diagram, it is seen that the curve is concave the temperature axis over the whole range of the observations, and therefore only effective magneton number which varies considerably with the absolute temrature T can be calculated.

The metallic acetylacetones of the general formula $M(CH_3.CO.CH.COCH_3)_3$, which M is any tervalent metal, form a series of stable readily crystallizable mpounds. Although these compounds cannot be regarded as simple salts, on count of their non-ionizability and their solubility in organic solvents, the metallic n seems frequently to possess the same ground state as in the corresponding mple salts. Thus ferric acetylacetone(3) has the same magneton number 5·94 as e simple ferric salts, and it seemed worth while to investigate the magnetic operties of manganic acetylacetone over a wide range of temperature in the hope obtaining a reliable estimate of the magnetic moment of the Mn^{+++} ion.

§ 2. METHOD AND MATERIAL

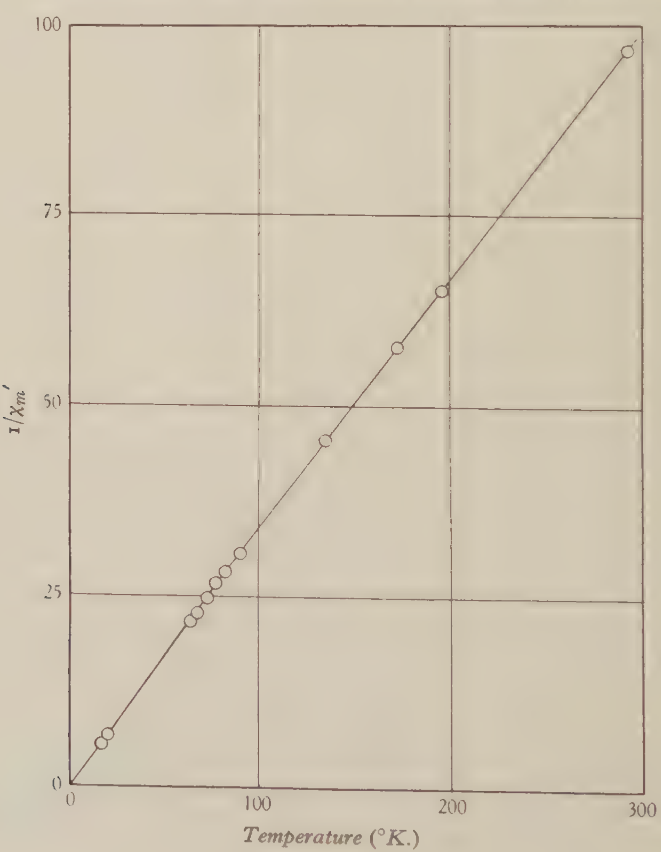
The susceptibilities were measured by the Faraday method with the aid of an paratus fully described in a previous paper(3). The specimen was sealed in a all spherical phial of pyrex glass and occupied the place of the crystal and carrier

shown in figure 1 of the paper in question. The low temperatures were obtaine
means of baths containing liquid ethylene, oxygen, nitrogen and hydrogen boi
under atmospheric or reduced pressure, and of a mixture of solid carbon dio
and acetone. The temperatures were determined by means of a copper-consta
thermocouple, or in the case of liquid oxygen and hydrogen, which were adequa
pure, from the vapour pressure of the liquid in the bath.

The manganic acetylacetone was kindly prepared for me by Dr R. C. Me
of the Chemistry Department of Bristol University. It was further recrystall
from acetone and a phial filled with the finely ground material. The substance
apparently in its monoclinic form(4) but this observation was not checked by ac
goniometric measurements.

§ 3. EXPERIMENTAL RESULTS

The specific susceptibility of manganic acetylacetone was measured from ro
temperature down to 16°·95 K. with the results given in the accompanying ta
χ_m' The reciprocal of the molecular susceptibility χ_m', corrected for the diamagne
of the organic groups in the molecule, is plotted against the absolute tempera
in the figure. It will be seen that the substance follows the Weiss law $\chi\,(T+\Delta$



stant with Δ equal to $+5{\cdot}5$ down to about 70° K. Small deviations from this occur at lower temperatures, the susceptibility increasing more rapidly with T is given by $\chi\,(T+5{\cdot}5)=$ a constant.

Table. Manganic acetylacetone

T (° K.)	$\chi\times10^6$	χ_m'	$1/\chi_m'$
292·2	$28{\cdot}8_8$	$0{\cdot}0103_5$	$96{\cdot}6_6$
196·0	$43{\cdot}1_0$	$0{\cdot}0153_6$	$65{\cdot}1_0$
172·7	$48{\cdot}1_8$	$0{\cdot}0171_4$	$58{\cdot}3_4$
135·2	$61{\cdot}9_0$	$0{\cdot}0219_7$	$45{\cdot}5_2$
90·2	$92{\cdot}1_5$	$0{\cdot}0326_3$	$30{\cdot}6_6$
82·3	100·0	$0{\cdot}0353_9$	$28{\cdot}2_6$
77·4	105·4	$0{\cdot}0372_9$	$26{\cdot}8_2$
72·7	113·6	$0{\cdot}0401_8$	$24{\cdot}8_9$
67·4	123·5	$0{\cdot}0436_7$	$22{\cdot}9_0$
63·9	129·6	$0{\cdot}0458_1$	$21{\cdot}8_3$
20·3	425·5	0·1500	6·66
$16{\cdot}9_5$	510·5	0·1799	5·56

§ 4. DISCUSSION OF RESULTS

The magneton number calculated in the customary way from the straight part the curve is 4·98 Bohr magnetons. It is thus quite comparable with that given Weber's work on solutions.

According to the theory of Bose-Stoner as worked out in detail later by Van Vleck magneton numbers of the ions of the elements of the first transition group ould lie between $\sqrt{\{4S\,(S+1)\}}$ and $\sqrt{\{4S\,(S+1)+L\,(L+1)\}}$, that is to say tween the value for spin only and that for spin and orbital moment both effective t uncoupled from one another. In the first half of the group the L contribution almost completely suppressed and the observed magneton numbers agree closely th the spin values. For the Mn^{+++} ion, the ground state of which is ${}^5D_0$ from und's rules, the magneton number should lie between 4·90 and 5·48 but approxi-ate closely to the first of these values. It will be seen that this is the case for the lue deduced above from manganic acetylacetone.

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THE COLOUR TEMPERATURES OF THE HEFNE AND ACETYLENE FLAMES

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ABSTRACT. The colour of the "cylindrical" acetylene flame depends on three fact namely the gas-pressure, the rate of consumption and the height of the flame. Discor statements in the literature are believed to be due to lack of uniformity in these particu The flame specified by Dr Jones of the Kodak laboratory is 50 mm. high and consu 0·25 ft^3/hr. at 90 mm. (water) pressure. A 1-cm. section of a flame conforming clo to this specification, burning on a Bray Vika tip, is found to have colour tempera 2520° K. The flame on a Bray Ota tip at 90 mm. pressure is 58–60 mm. high and consu 0·29–0·30 ft^3 hr. One cm. sections of two such flames were found to have colour t peratures (in round figures) 2380° and 2400° K. We have not been able to test a l enough number of burners to say what limits of variation may be expected, but we bel that if the three particulars mentioned above are closely adhered to, either flame is re ducible within a maximum range of ±20° from the mean.

The colour temperature of the Hefner flame is found to be 1910° K.

Although now somewhat out of fashion the Hefner lamp is still for m purposes a useful photometric standard and the cylindrical acetylene fl has been repeatedly recommended as a secondary standard(1). The spec distribution of the light is however subject to doubt in both cases, as the follow summary of published work shows.

The Hefner flame. Tumlirz(2) made a spectrophotometric comparison over range 0·43–0·7 μ. between the Hefner radiation and sunlight, for which he assu Langley's data. The resulting Hefner spectrum corresponds very roughly wit Planckian curve for 2100° K. Angström(3) found by bolometric measurem from 0·50 to 0·75 μ. that the colour temperature was 1830° K., and Leder(4 spectrophotometric comparisons against black radiators at 1452° and 1616 confirmed Angström's result. Hyde and Forsythe(5) by comparison of tints Lummer-Brodhun head against standard filament lamps of known {voltage, col temperature} characteristics ascribe to the Hefner flame a colour temperatur 1880° K. Valentiner and Rössiger(6) by means of a monochromator and ph electric cell made a spectrophotometric comparison of the brightest part of Hefner flame and the electrode of a tungsten arc lamp, and concluded that spectrum could be represented by a Wien equation for 1840° K. Dziobek Hoffman(7) compared the Hefner flame with a black radiator both spectroph metrically and by matching tints, and found its colour temperature to 1900 ± 20° K. by the first method, and 1910 ± 5° K. by the second. The v most generally accepted appears to be 1875–1880° K.

The acetylene flame. According to Coblentz and Emerson(8) the flat (fishtail) etylene flame is not sufficiently reproducible to be standardized, but the cylindrical r rather cigar-shaped) flame gave "sufficiently concordant results to warrant its e as a standard in spectrophotometric work not requiring the highest precision". uild(9) gives it a similar qualified approval. For photometric purposes Mees and eppard(1) and Jones(1) used a Bray burner consuming $\frac{1}{4}$ cubic foot of acetylene r hour with a flame 35 mm. high (Mees) or 50 mm. high and 3 mm. thick (Jones). ariation of the gas pressure from 8 to 10 cm. of water caused only 2·7-per-cent ange in the photometric intensity of Jones's flame. Coblentz and Emerson amined radiometrically the spectra of the middle sections of two acetylene flames ch consuming $\frac{1}{4}$ cubic foot per hour; the first, on a Crescent Aero tip, measured 5 mm. by 3 mm. under 7·5 cm. pressure; the second on a Bray tip was 50 mm. gh at 9·0 cm. pressure. The spectra were identical throughout the visible region far as 0·7 μ., beyond which wave-length the Bray flame began to show the higher ntensity. Subsequent calculations(10) showed that the experimental values agreed ithin the limits of error (± 1 to 2 per cent) with a Wien curve for 2360° K. except the blue and violet beyond 0·48 μ., where they began to grow higher. Hyde, orsythe and Cady(11) sent tungsten filament lamps, which had been standardized gainst black radiators, to the Eastman Kodak laboratory and to the Bureau of tandards, with a request that they should be colour-matched with an acetylene ame. The Kodak laboratory using a Bray tip at $\frac{1}{4}$ ft.^{3}/hr. found a matching voltage hich gave a colour temperature of 2352° K.; the Bureau of Standards using the ero tip found at first 2434°, and on re-test 2450° K. Hyde, Forsythe and Cady hemselves made both colour-match and spectrophotometric comparison with their andardized lamps, and concluded that the Bray tip operating at 9 cm. pressure ave a flame of colour temperature 2360 ± 10° K. Lowering the gas pressure to ·5 cm. changed this figure only a few degrees. Davis and Gibson(12) found that a perfect colour-match between the acetylene light and the unfiltered electric ight could not be obtained at any colour temperature". The nearest approach was omewhat over 2400° K. They point out that Coblentz's spectral energy values orrespond, by the correlation method of Davis(13), with a colour temperature of 413° K.; and by that of Priest by way of the spectral centroid, with 2388° K. The value 2360° recommended by Hyde, Forsythe and Cady is commonly accepted, ut the above discrepancies clearly need an explanation. The confusion confronting nquiry into the literature of this subject is no doubt responsible for a misleading able on p. 401 of volume 19 of the *Handbuch der Physik* of Geiger and Scheel, which purports to give the spectral-energy ratios of the acetylene flame to black radiators at 2360° and 2450° K. The figures in this table were in fact calculated by Hyde, Forsythe and Cady to show that the uncorrected radiometric results first published by Coblentz did not correspond with the spectrum of a black body at any temperature.

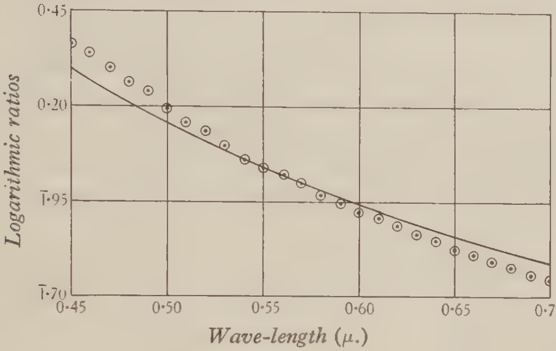
During the past two years we have compared a cylindrical acetylene flame with a Hefner flame on a Hilger-Nutting spectrophotometer. In all seven sets of readings, each duplicated by two observers, have been taken. Corresponding sets of readings of the instrumental constants were taken, and it may be remarked t the flame is very suitable for this purpose on account of its axial symmetry. T procedure was that one observer made at least four optical settings at each wa length, starting at 0·57 μ., progressing alternately towards the red and the vio by steps of 0·02 μ., and returning similarly by way of intermediate wave-lengths, as to give a complete set of readings at 0·01-μ. intervals from 0·45 to 0·70 μ. T settings were read off by a second observer in order to preserve the sensitiv of the main observer's eye.

The acetylene was generated from commercial carbide (in accordance with practice of Jones, Coblentz, Davis, etc.) and between 98 and 99 per cent of it absorbed by bromine water. It was passed through towers containing respectiv copper-sulphate crystals and anhydrous calcium chloride. Hydrides of phospho are always present in carbide-generated acetylene, but not in sufficient amount affect the spectrum appreciably. No change could be detected in spectrogra from acetylene to which quantities of phosphine up to 1 per cent by volume been added. The gas was stored overnight before use in a 5-ft^3 holder over wat and was burnt on a Bray Vika tip taking $\frac{1}{4}$ ft^3 hr at a pressure of 9 cm. of water. T holder bell had compensating weights whereby the pressure was automatically k within a range of ± 1 mm. The flame was from 49 to 50 mm. high and about 3 m thick in its central portion. A hood of the form described by Jones shielded fr observation all except the central section from 15 to 25 mm. up the flame. T whole Hefner flame was observed in the other pupil of the spectrophotome The collimator slit was 0·5 mm. wide. The precision attainable with the Hilg Nutting instrument has been considered in detail recently by Twyman Lothian(14) and our experience leads us to think that they have certainly not ov estimated it. The average deviation of a single density reading from the arithm mean of its group was from 0·005 to 0·01 for wave-lengths from 0·48 to 0·66 μ., increased to about 0·03 at 0·45 μ. Introduction of filters to stop stray light at extreme wave-lengths made no detectable difference to the settings. The wa length scale of the spectrometer was checked at a number of mercury and ne lines, and the density scale of the photometer was checked by means of a sec reading to 1 min. of arc.

The mean experimental values of the spectral ratios acetylene : Hefner, adjus by a constant factor to unity at 0·57 μ., are given in column (2) of table 2, page 10 and their logarithms are plotted in figure 1 alongside those calculated from accepted colour temperatures 1875 and 2360° K. The slight inflections in experimental curve at wave-lengths 0·49, 0·54–0·55 and 0·58–0·60 μ. are reprodu with great fidelity in all the tests. They do not appear to be an instrumental eff because, apart from the fact that they remain after allowing for the instrume constant, they are not altered when the two flames are interchanged in the set-

The clear discrepancy between experimental and calculated ratios led us compare each flame separately with the anode of a tungsten arc lamp which co be kept at a constant determinable temperature.

The arc was a pyrometric Pointolite having a bun-shaped anode about 8 m diameter, and the temperature of the outer surface of the anode was read on a ambridge disappearing-filament pyrometer immediately before and after each mparison, the current being held constant within narrow limits by manual ljustment during the whole period. It was first necessary to see if the temperature



igure 1. Logarithms of (acetylene : Hefner) ratios. ——— calculated for 1875° and 2360° K. ⊙ experimental comparison.

f the tungsten surface varied locally, and therefore an extension was fitted to the yrometer tube so that different portions of an enlarged image could be observed. able 1 gives uncorrected pyrometer readings taken (*a*) in the centre of the enarged image; (*b*) about $\frac{2}{3}$ of a radius above the centre; (*c*) and (*d*) at similar disnces along horizontal radii towards the sides nearest respectively to the lighting oil and the main cathode rod.

Table 1. Temperature-distribution over surface of tungsten anode

Position of arc on anode	Side	Centre
Current (A.)	0·95	1·00
Temperature (° K.) observed at (*a*) centre	1459	1477
(*b*) top	1458	1469
(*c*) coil side	1484	1472
(*d*) rod side	1442	1467
Weighted mean	1462	1474
Temperature observed on normal unenlarged image	1462	1479

The arc strikes at the side of the anode nearest the lighting-coil, and occasionally t small currents it fails to move over into the centre. The first column of figures hows that in this position there is a temperature-gradient from the coil side to the od side. A vertical central strip of uniform temperature with hotter and cooler reas at its respective sides being assumed, a mean value, weighted as to area and brightness, was calculated and was found to agree with temperatures read from the small image when the extension tube of the pyrometer was not used. By temporarily increasing the current the arc could be made to travel to the centre of the anode, but there was a tendency to wander when the arc was run for long periods at a low current, and therefore the results at lower temperatures are considered less reliable than those at higher temperatures. When the arc was central on the anode th slight radial temperature-gradients were within the limits of reproducibility fro day to day of the pyrometer readings. In actual working, therefore, the arc wa always manoeuvred so as to settle in the centre of the back of the anode and th pyrometer was used without extension tube. A stop was fitted close to the bul enclosing the arc, whereby all radiation from the lighting-coil and cathode ro (which project from behind the anode at the two sides) and from the lower edge the anode where its supporting rod is connected, was shielded from the spectro meter. The pyrometer, which had been checked at the National Physical Laborator had a red glass filter, the transmission of which was determined. Its effective wav length was 0·6475 μ. at 1910° K. The readings were corrected by means of Wien equation for loss of brightness due to the glass of the bulb. For this purpose th bulb was broken at the end of the experiments and spectral transmission factor given in column 6 of table 2, were determined on the area which had been expos by the stop above mentioned. In calculating the overall loss, re-reflection at th surface of the anode was taken as 56 per cent from consideration of data by Coblentz(1 for cold tungsten and by Langmuir(16) for molten tungsten. The actual brightne temperature of the anode surface having been thus determined, the correspondin colour temperature was calculated from the tables of Forsythe and Worthing(1 Forsythe(18) has shown that the emission of tungsten is very nearly that of a blac body at the same colour temperature, and we therefore assume, as the basis our comparisons, energy curves calculated by Wien's equation for the determin colour temperatures.

The comparisons of the two flames against the tungsten arc were made in tw ways, (*a*) photographically with spectrometer alone, and (*b*) visually with spectro photometer.

(*a*) With exposures suitable for the blue end of the spectrum, the green an yellow are so over-exposed that the spectra can only be compared over a sma range. We therefore inserted in the camera just in front of the plate a filter coverin the spectral region from about 0·54 μ. onward, whereby the intensity of that pa was reduced to about the same photographic intensity as that of the blue en Suitable filters were, for the acetylene flame, Wratten No. 78 AA; and for the Hefn flame, Wratten No. 37 in double thickness. The method of making the compariso was then the following. A series of equal exposures of the flame were made, so th the spectrograms were spaced at equal intervals from top to bottom of the plat This was easily done by means of a graduated rack motion fitted to the camer The flame was then replaced by the tungsten arc running at some selected curren A rotating sector was interposed whereby the illumination was reduced to intensity rather greater than that of the flame in the previous exposures. A seri of spectrograms of the arc was then taken in the intervals left on the plate betwe the flame spectrograms, and at each exposure the intensity of the illuminati reaching the spectrometer was reduced in 5-per-cent steps by means of the secto Under these conditions, if the arc happened to have the same colour temperatu as the flame the first pair of spectrograms would show the arc spectrum uniform deeper than the flame spectrum; the last pair would show it uniformly lighter than the flame spectrum; and there would be some intermediate pair which would match each other exactly throughout the spectrum. If, however, the arc had a colour temperature differing from that of the flame, one end of its spectrum would be reduced to equality with the flame spectrum sooner than the other, which would require one or more further stepwise reductions of the sector aperture before it matched. The number of steps between the matching of the two ends of the spectrum is thus a measure of the difference in colour temperature between arc and flame. Such sets of paired spectrograms were taken with the arc held at a series of temperatures, and for each temperature the number of steps measuring the difference between the matching pair for the red and that for the blue end was noted. The differences were plotted against arc temperatures, figure 2, and the point at which the best-fitting line cut the temperature axis was taken as that at which the two colour temperatures were identical.

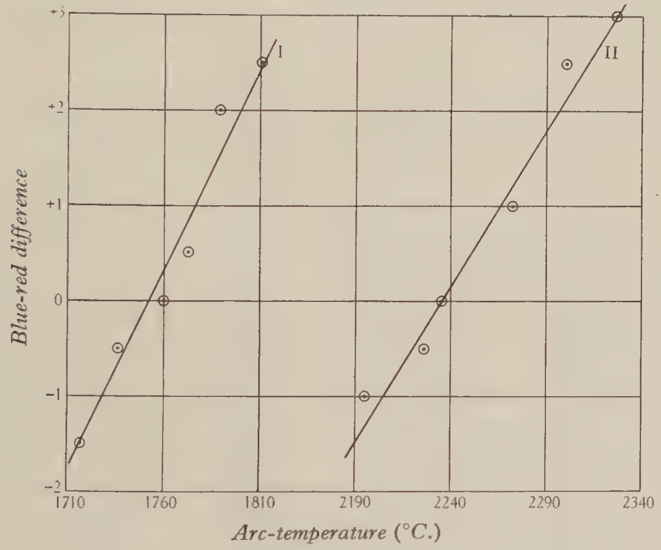


Figure 2. Curve I, Hefner; curve II, acetylene.

In this way the Hefner flame was found to match the arc running at a pyrometer reading of 1752° K. This, corrected for the effect of the glass bulb, gives the brightness temperature 1758° K. which, from Forsythe's tables, corresponds with a colour temperature of 1913° K. The acetylene flame was found similarly to match a colour temperature of 2519° K.

(*b*) The above photographic method takes no account of any possible selective transmission by the glass of the arc bulb. Such selectivity is very slight, as may be seen from the transmission factors in table 2, column 6, and its effect is allowed for in the following visual comparisons of each flame against the arc, made on a Hilger-Nutting spectrophotometer. The settings were made as already described by each of two observers, the other meanwhile controlling the arc current and taking down the instrumental readings. The arc was run at a current correspondin with a colour temperature of 2520° for the acetylene flame, and 1925° K. for th Hefner flame. The spectral factors averaged from four sets of readings by eac observer were corrected in respect of the transmission factors of the bulb glass an then applied to spectral-energy values calculated from Wien's equation for th appropriate temperature. The results giving the spectral distribution of the radian energy from each flame are tabulated in columns 3 and 5 of table 2, while column shows the Hefner spectrum calculated by means of factors obtained from direc

Table 2

1 Wave-length (μ.)	2 Ratio of acetylene to Hefner, by direct comparison	3 Acetylene flame	Hefner flame by comparison with		6 Trans-mission factors of bulb glass
			4 Acetylene flame	5 Tungsten arc	
0·72	—	257·6	—	—	—
0·71	—	246·0	—	—	0·914
0·70	0·561	234·4	407·4	—	0·916
0·69	0·573	221·6	381·5	384·1	0·918
0·68	0·594	209·4	349·9	352·4	0·918
0·67	0·621	198·2	318·4	323·6	0·920
0·66	0·649	187·1	289·1	294·4	0·920
0·65	0·668	177·0	265·2	267·9	0·923
0·64	0·705	165·8	236·9	240·7	0·923
0·63	0·734	155·2	213·8	214·3	0·923
0·62	0·769	145·7	191·2	190·7	0·923
0·61	0·807	137·1	169·8	169·4	0·923
0·60	0·836	127·2	152·6	151·2	0·923
0·59	0·879	117·2	134·3	133·0	0·923
0·58	0·925	108·2	117·6	115·7	0·923
0·57	1·000	100·0	100·0	100·0	0·923
0·56	1·055	91·3	86·5	86·2	0·923
0·55	1·105	82·7	75·3	74·6	0·920
0·54	1·156	74·9	64·9	63·2	0·918
0·53	1·253	67·3	54·0	52·8	0·916
0·52	1·365	60·0	44·4	43·9	0·916
0·51	1·439	53·2	37·4	36·3	0·914
0·50	1·560	46·9	30·5	29·9	0·914
0·49	1·742	41·3	23·9	24·4	0·916
0·48	1·826	36·2	19·9	19·7	0·918
0·47	2·004	31·3	15·6	15·7	0·918
0·46	2·183	26·9	12·3	12·5	0·918
0·45	2·307	22·9	9·9	9·8	0·916
0·44	—	19·4	—	—	—
0·43	—	16·2	—	—	—
Colour temperature (° K.)		2520	1912	1897	—

comparison of the two flames, on the assumption that the acetylene flame radiat as a black body at 2520° K. At the foot of each column is the black-body temperatu having the same centroid over the range 0·45–0·69 μ. as the tabulated spectru In figure 3 the experimental values from columns 3 and 4 have been plott alongside Wien curves for 2520° and 1910° K. respectively. The figures in colum lead to a slightly lower colour temperature than those in column 4, and they are considered to be less reliable on account of the tendency of the arc to wander occasionally when run at low currents.

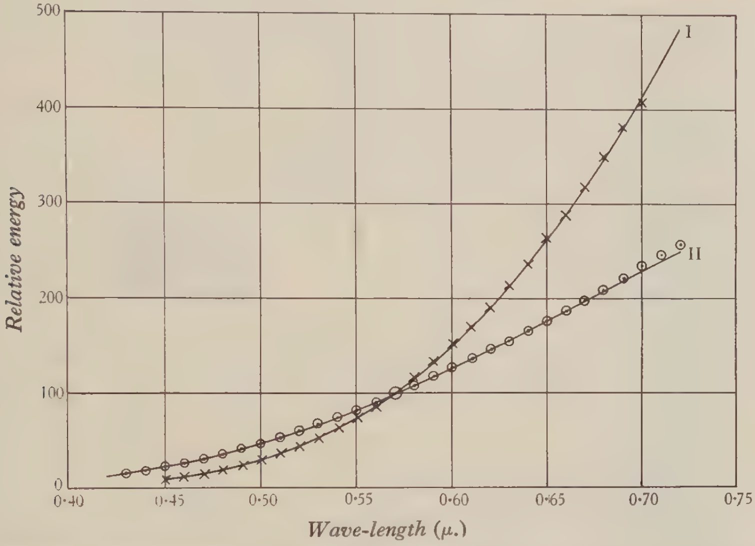


Figure 3. Curve I, black body at 1910° K.; × Hefner flame. Curve II, black body at 2520° K.; ⊙ acetylene flame.

The values given in table 3 for the colour temperature of the entire Hefner flame are higher than the accepted value 1875–1880° K.; they agree with the estimates of Dziobek and Hoffmann referred to above.

Table 3. Summary of colour temperature determinations

Method	Hefner (° K.)	Acetylene (° K.)
Photographic comparison against tungsten arc	1913	2519
Spectrophotometric comparison against tungsten arc	1897	2520
Spectrophotometric comparison against acetylene flame	1912	—

The colour temperature of the central section of our acetylene flame is much above any other published value, and we sought an explanation of the discrepancy by examining the flames produced by various burners. The burners are manufactured by Geo. Bray and Co. Ltd., of Leeds, in two types under the trade names Ota and Vika, and in sizes consuming $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{2}$ cubic feet per hour. The makers state that the consumption figures stamped on the sockets show the hourly consumption of gas at a pressure of 3 in. of water. Previous workers have used pressures of 75 and 90 mm. of water, and we therefore tested the consumptions of several burner tips at these pressures and compared some of the flames spectrophotometrically with that used in the tests described above, using always the central portions of the flames at heights of 1·5–2·5, 2·5–3·5 and 1·0–1·5 cm. respectively above the bases of the Vika $\frac{1}{4}$-ft^3, Ota $\frac{1}{4}$-ft^3 and Vika $\frac{1}{8}$-ft^3 flames. Table 4 contains the results. The consumptions were measured by means of a water-sealed meter passing $\frac{1}{12}$-ft^3 per revolution and calibrated with the $\frac{1}{12}$-ft^3 bottle as prescribed by the Gas Referees. They are given in cubic feet per hour measured at 60° F. and 30 in. of mercury. The heights of the flames were measured in a darkened room since otherwise the faintly luminous tip might be overlooked.

Table 4. Details of Bray burners

Maker's description		Consumption at 75 mm. (ft^3/hr.)	Consumption at 90 mm. (ft^3/hr.)	Flame height at 75 mm. (cm.)	Flame height at 90 mm. (cm.)	Colour temperature at 75 mm. (° K.)	Colour temperature at 90 mm. (° K.)
Vika $\frac{1}{4}$-ft^3	*(a)*	0·22	0·245	40	48–49	2535	—
	(b)	0·22	0·25	41	49–50	—	(2520)
Vika $\frac{1}{8}$-ft^3	*(a)*	0·11	0·127	18	22	—	2670
	(b)	0·11	0·126	19	21–22	—	—
Ota $\frac{1}{4}$-ft^3	*(a)*	0·27	0·30	52–53	60	—	—
		0·25	0·28	48–49	54–55	—	—
	(b)	0·26	0·29	52	59–60	2377	2407
	(c)	0·26	0·30	52	59	—	—
	(f)	—	0·29	—	58	—	2377

The results show that the small ($\frac{1}{8}$-ft^3) Vika flame is whiter, and the Ota $\frac{1}{4}$-ft^3 flame is yellower, than the $\frac{1}{4}$-ft^3 Vika flame. The differences are clearly attributable to differences in primary aeration. An acetylene burner must admit enough primary air to stiffen the flame and prevent carbon-deposition. It is well known that as the primary aeration is increased the flame-temperature rises, so that the flame grows continuously whiter although its total luminosity soon reaches a maximum and begins to fall. Thomas[(19)] has shown that, other things being equal, the volume of air entrained per unit volume of gas delivered increases as the size of the jet diminishes; and the colour temperatures found are in fact in reverse order of the consumptions, thus: Vika $\frac{1}{8}$ > Vika $\frac{1}{4}$ > Ota. Thomas also showed that air-entrainment increases with delivery pressure, and this is exemplified in the case of Ota burner (*b*), the colour temperature of which is raised by increasing the pressure from 75 to 90 mm.

The majority of previous workers used burners consuming $\frac{1}{4}$-ft^3 hr. at a pressure of 90 mm., with a flame 50 mm. high. The Vika tip which we have used as our standard conforms nearly to this specification. If these three easily measured magnitudes—gas-pressure, consumption and height of flame—are fixed, it appears that the flame is sufficiently reproducible for use as a spectral standard when the highest accuracy is not required. It is however necessary to select the tips carefully. For example, the Vika $\frac{1}{4}$-ft^3 tip (*a*) in table 4 is slightly under size, and has a slightly but distinctly higher colour temperature than the standard (*b*). Some tips are not quite truly set in the socket, and give flames which are not vertical. The dimensions of the primary airholes are not very critical, at any rate within certain limits. Our standard Vika tip has four holes each of diameter 1·2 mm. We selected two tips having the same consumption and flame height, closed up one of the four holes of

Figure 4. Unenlarged synchronous spectrogram of moving striations in neon in orange and red regions.

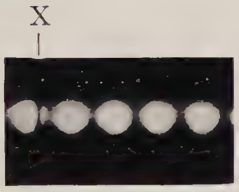


Figure 5. Moving striations in neon.

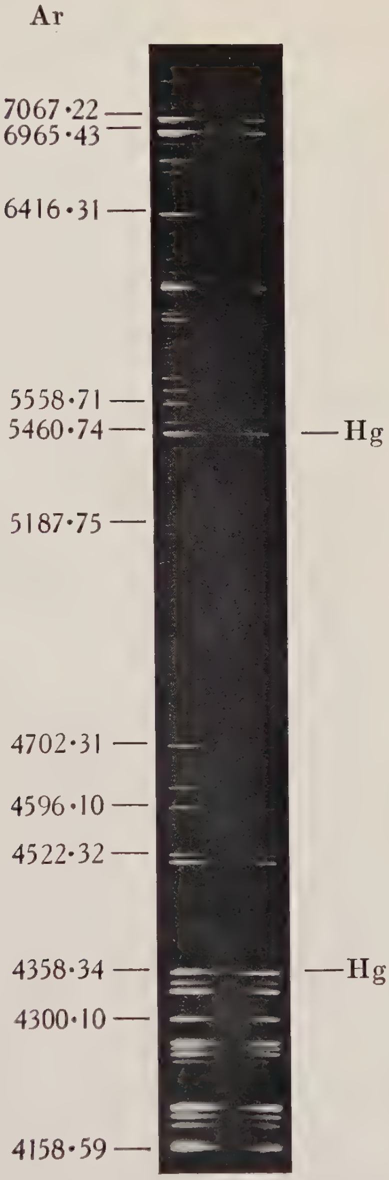


Figure 7. Synchronous spectrogram of striations in argon-mercury mixture.

the first, and enlarged those of the other to 1·3 mm. diameter. On spectrophotometric comparison with the standard the former gave a colour temperature of 2522° K., which was indistinguishable from that of the standard, and the latter 2527° K. Variations so great as these intentional changes of size would not be met with in practice. The airholes can in any case be easily gauged by means of standard drills.

The Ota tips approach their rated consumption only at the lower pressure. Hence their flame, as has already been stated, is larger, less aerated and cooler than that of the Vika tip. Their airholes are also slightly smaller. The Ota tip (*a*) in the table possessed two definite consumption-rates. This phenomenon is possibly due to some irregularity in the bore causing instability of flow, but as the temperature of the gas in the bore is unknown it is impossible to say definitely that its velocity falls within the critical range. The tip (*f*) was kindly lent by Dr Forsythe from his own apparatus. Our value for the colour temperature of this flame is slightly higher than that given by Dr Forsythe, which was 2360° K. Ota tip (*b*) gave a flame of 2407° K. when tested over a length of 1 cm. With slot reduced to expose only about 4 mm. the result was 2402° K. The difference is within the limits of variation of individual tests.

ACKNOWLEDGMENTS

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THE ELECTRON-OSCILLATION CHARACTERISTICS OF AN EXPERIMENTAL PLANE-ELECTRODE TRIODE

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ABSTRACT. To investigate the characteristics of electron oscillations generated und simplified conditions, an experimental plane-electrode triode has been constructed. It plane emitting surface is 25 mm. in diameter. The cathode-plate distance can be continuously varied while the valve is in operation. External circuits may be connected to th electrode leads in such a way that the resultant oscillating circuit is effectively continuou from the electrodes to the terminating condenser. With no external circuit connected t the valve, oscillations are maintained whose wave-length varies with grid voltage exactl according to the Barkhausen equation $\lambda^2 V = \text{constant}$. Their mechanism appears to b confined to the grid-cathode space. They are produced only when the grid current i space-charge-limited. When an external circuit is used, oscillations are maintained a very low emissions. Their wave-length is always identical with some resonant wave-length of the circuit. Their mechanism may be confined to the grid-plate space, or ma extend over the cathode-plate distance. First and second-order dwarf waves are observe It is shown that much more complex experimental results can be analysed on the bas of the conclusions drawn from the simplified experiments. A brief discussion of th applicability of existing theories to the simplified experiments is given.

§ 1. INTRODUCTION

In recent years a very large amount of experimental work(1) has been carrie out in the investigation of the mechanism and characteristics of electro oscillations. A satisfactory solution of the problems involved has not, howeve been found. The phenomena observed have been so complex and so diverse that has not been possible to penetrate through them to common fundamental law The many theories which have been advanced are based on a wide variety mechanisms and, while each has had some degree of experimental justification, the have been numerous instances in which investigators have been unable to duplica the results upon which others have founded such theories.

Consideration suggests that this confused situation can only be due to the fac that, with a few minor exceptions, the experiments have been made entirely wit commercial valves, and that different experimenters have used different types valve. Valve-construction parameters play a very critical part in the generation electron oscillations. Potapenko(2) and others have shown that even valves superficially identical may have quite different oscillation characteristics. Further, th structure of the electrodes and the electrode leads in commercial valves is not amenable to simple theoretical treatment. Hence the results that have been obtained cannot in general be compared directly with one another or with theoretical results based on simplifying assumptions.

In the present investigation an attempt has been made to reduce the problem experimentally to its simplest terms. A plane-electrode experimental valve has been designed to approximate as closely as possible to the theoretically simplest arrangement, and to eliminate indeterminate geometrical parameters. The aim has been to see whether the phenomena observed under these idealized conditions are explainable by any simple theory.

In this paper there are described only the basic results that have been obtained, with the number of critical variables reduced to an absolute minimum. Much more complex effects are observed when other variables are allowed to become important. Operating values for these minor variables have been so chosen as to make their influence either negligible or easily calculable. The results described have all been reproduced several times in essentially the same form, if not in exact detail, and it is claimed that they can be repeated by any experimenter.

§ 2. THE EXPERIMENTAL APPARATUS

The details of the experimental valve used are given in figure 1 (*a*). The Pyrex envelope *P* is sealed by white wax into a circular slot a few millimetres deep in the brass ring *B*. A circle of copper tubing soldered into another slot in the ring permits water cooling of the wax seal. The square of plate glass *D*, through which the leads to the electrodes pass, is sealed with tap grease to the polished face of the ring *B*. Plasticene around the circumference of the ring completes the seal.

The electrode leads are nickel rods 3 mm. in diameter. They are threaded for 2 in. of their length at the ends nearest the electrodes, and are fastened to the glass *D* by a washer and nut on either side. White wax is used as seal. Two leads are required for the cathode-heating coils and one each for the three electrodes. The grid lead has two adjustable joints, so that the grid-cathode distance can easily be varied on dismounting the valve.

The plane anode (which will be referred to henceforth simply as the plate, since it is not in this work a positive electrode) is fixed to a short length of Pyrex tubing which slides vertically in the tube *T*. A silk-thread suspension allows the vertical height of the plate to be varied by rotation of the rod *R* through the ground joint. Motion through a distance of 2–3 cm. is possible. The plate lead consists of a short length of nickel tube 3 mm. in inside diameter, pivoted at the plate, and telescoping on to a piece of 3-mm. rod pivoted just inside the glass *D*. With this construction the glass *D* carrying the grid and cathode can be dismounted, leaving the plate in position. Inter-electrode distances are measured with a microscope sliding on a vertical rod outside the envelope.

The constructional details of the indirectly-heated plane cathode are shown in figure 1 (*b*). The component parts are all mounted on a small soapstone base which provides the insulation between the heating supply and the cathode surface. Ther are two heating-coils, each requiring about 15 cm. of tungsten wire 0·5 mm. i diameter, wound into 20 turns 2·5 mm. in mean diameter. The box containing th heaters is circular, 25 mm. in diameter, and formed from nickel sheet of thickne 0·5 mm. Similar sheet is used for the two radiation shields. The upper edge of th cathode box is spread to form a flange, and a circle of 0·1-mm. nickel foil is spot welded to this flange to form the emitting surface. A suspension in water of finel

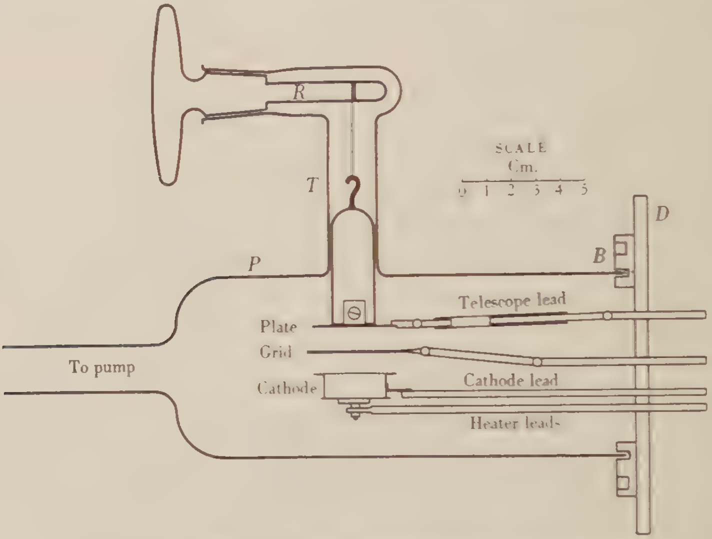

Figure 1 (*a*). The experimental plane-electrode valve.

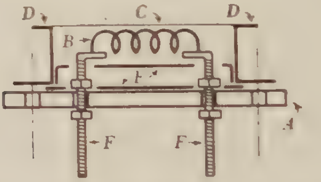

Figure 1 (*b*). Constructional details of the plane cathode. *A*, soapstone base; *B*, tungsten heatii coils; *C*, emitting surface; *D*, non-emitting guard ring; *E*, radiation shields; *F*, current leads.

ground barium hydroxide painted on to the foil, and dried in air, gives emissio very easily and quickly, provided that the surface is clean and the system we sealed. The 5-mm. flange around the actual emitting surface serves as a guard rin to eliminate any possible distortions of the electrostatic field that might be cause by the electrode leads, or other edge effects.

The grid used throughout the experiments described in this paper consists c parallel 0·5-mm. nickel wires spaced at 1·5 mm. between centres, spot-welded a either end to a 1·5-mm. nickel wire frame. The frame is 4 cm. square. Gric transparency (67 per cent in this case) has been found experimentally to be a ver important parameter, and the results described here are not in every case cha cteristic over a wide range of this variable. The plate used throughout consists of circle of sheet nickel 3·5 cm. in diameter.

The parallel-wire external circuits which can be connected to the valve are all onstructed of brass telescope tube, of intermediate diameter 7 mm. Connexion to e electrodes is made by threading the smaller brass tube over a short length of rass rod which in turn threads over the nickel electrode leads. The spacing between e two conductors is always the same as that between the electrode leads where hey pass through the glass plate *D*, whichever pair of electrodes is being used. The rminating condensers are Sangamo 0·0005-μF., and are fastened to very short ngths of brass strap soldered to the ends of the brass tubes.

With this arrangement the resonant circuit in which oscillations may occur onsists effectively of parallel rods extending continuously from the electrodes to e terminating condenser. It can thus be accurately analysed and the indeterinacy introduced by the electrode-seal in commercial valves is eliminated. The hange in circuit constants just outside the glass plate is not serious, and no inication of spurious reflection from this point has ever been obtained.

The leads carrying the heating-current to the valve from the transformer are onnected by a 0·001-μF. condenser immediately outside the plate *D*, and by two ther similar condensers spaced at 15 cm. along the length of the leads away rom the valve. This effectively prevents the leads from acting as a resonant circuit t any of the frequencies generated experimentally. Radiofrequency chokes, used herever necessary, are space-wound of 32-gauge wire on 5-mm. glass tubing, and ulfil their purpose satisfactorily.

By detecting oscillations generated by this valve with a heterodyne receiver, t has been shown that the magnetic field produced by the cathode-heating coils as no observable influence on the nature of the oscillations. When the heating urrent is turned off the cathode remains hot enough to emit for several seconds, nd the detected oscillations remain unchanged in frequency and intensity for at east 1 or 2 sec., before they gradually decay away. Nickel loses its ferromagnetic roperties at a temperature far below that at which the cathode normally operates, o that no effects can be attributed to magnetization of the cathode box.

For measuring wave-length, absorption wave-meters similar to those described y Moore(3) have been calibrated down to 60 cm. Where these cannot be used, a rystal circuit of the type described by Chapman(4) is very loosely coupled to the alve. This circuit may also be used to give an indication of oscillation-intensity. he coupling is so loose that resonance in the crystal circuit does not visibly affect he plate current of the oscillator. There can be no question of appreciable interction between the two circuits.

§ 3. THE EFFECTS OF GAS PRESSURE

Nettleton in 1922 published a paper(5) describing "electron" oscillations which isappeared when the pressure in the valve was reduced to $5 \cdot 10^{-5}$ mm. of mercury. his immediately suggested that the oscillations were not produced solely by

electrons, but required the presence of positive ions. Tonks and Langmuir$^{(6)}$ ha
shown experimentally that an ionized gas can produce oscillations of the corr
order of frequency. The pressure required to give a sufficiently high ionic dens
must be about 10^{-3} mm. It is possible, therefore, that at such pressures the posit
ions do play an essential part in the generation of oscillations in a positive-g
triode.

For the usual cases of oscillations in positive-grid circuits, however, it has be
proved very conclusively by Grechowa$^{(7)}$ and by Rindfleisch$^{(8)}$ that the princi
effect of gas is to damp the oscillations. These workers found, in independ
experiments, that the oscillation-intensity increased asymptotically to a maxim
as the pressure decreased down to 10^{-8} mm.

Before experiments were started with the present plane-electrode valve
pressure test similar to those described in the above papers was carried out on
AT 40 valve. The object was to determine an upper limit for the pressure at wh
oscillation-intensity and wave-length can be regarded as effectively independent
the presence of the gas. The AT 40 is an ordinary commercial valve of cylindric
symmetrical construction, having an anode-diameter of 12·8 mm. and a gr
diameter of 5 mm. A glass tube was sealed into the valve at the tip, a small b
containing charcoal was joined to this tube, and the whole was sealed on to
diffusion pump. After baking of the glass, degassing of the electrodes by electr
bombardment, and heating of the charcoal for some time, the valve was calibra
as a positive-grid ionization gauge against a McLeod gauge for several values
pressure down to 10^{-5} mm., the lowest pressure readable with any accuracy on
McLeod gauge. After further heating and pumping the valve was sealed off
removed from the pump. Linear extrapolation of the calibration curve of
ionization gauge being assumed to be valid, it was then found that liquid air appl
to the charcoal would reduce the pressure in the valve to slightly below $2 \cdot 10^{-7}$ n
By surrounding the charcoal bulb with ice, boiling water, or a heated oil bath, ot
values of pressure could be maintained steadily. Values of optimum grid volta
wave-length, and oscillation-intensity were taken at each pressure, the exter
circuit remaining fixed as a three-quarter-wave system. The results are shown
figure 2.

These results are in general agreement with those previously mentioned. T
oscillations disappear at a pressure of about $5 \cdot 10^{-4}$ mm., owing probably to
electrons losing too much energy to the gas molecules. The considerable incre
of wave-length with pressure may be due to a combination of two effects.
inter-electrode capacity, which forms a part of the resonant circuit, varies with
ionic density between the electrodes, so that the resonant frequency of this circ
is being changed. The presence of the ions and molecules must also affect
electron transit time. The fact that the experimental grid-voltage curve falls m
more rapidly than the theoretical grid-voltage curve corresponding to the exp
mental wave-length curve suggests that the effect of the gas is to reduce the tra
time.

For these fairly intense oscillations it is seen that pressures up to 10^{-4} m o not affect the intensity appreciably, and affect the wave-length by only about per cent. Oscillations of very small intensity, however, generated at low emission urrent, disappeared when the pressure was increased to about $5 \cdot 10^{-5}$ mm., and heir wave-length at this pressure was about 4 per cent higher than at the minimum ressure. They remained unaffected for pressures up to $3 \cdot 10^{-5}$ mm. It may thereore reasonably be considered that at pressures below 10^{-5} mm. both oscillationntensity and wave-length may be regarded as independent of the gas, whatever the ctual magnitude of the oscillation intensity.

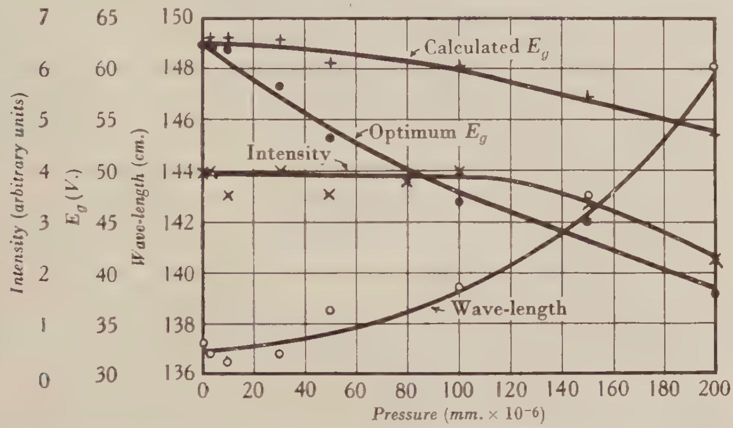

Figure 2. Effects of gas pressure on the wave-length and intensity of electron oscillations generated in a cylindrical-electrode triode. Filament current and external circuit-length constant.

Because of the numerous wax and glass seals in the plane-electrode valve, the ize of the electrodes, and the fact that the electrodes are exposed to the atmosphere each time the valve is dismounted, the attainment of a pressure very much below 10^{-5} mm. would require that a great deal of time be spent on checking seals and degassing the electrodes. The pump used is a steel mercury-vapour condensation pump, and the connexion from the pump to the valve consists of 20-mm. Pyrex tubing. This equipment with a liquid-air trap will maintain the pressure at about 10^{-5} mm. even when there are small leaks present, and will produce a much lower pressure if the system is perfectly sealed. It will be considered throughout, therefore, that gas pressure is not influencing the results to any measurable extent.

§ 4. BASIC ELECTRON OSCILLATIONS

The most fundamental generator for the production of true electron oscillations should be one which does not depend on any resonant inductance-capacity circuits. Oscillations obtained from such a generator must be regarded as being characteristic of some property of the electron motion alone, and not simply as oscillations inevitably produced in a resonant circuit of zero effective resistance.

Barkhausen and Kurz visualized oscillations of this type in their original paper on the subject⁽⁹⁾, although they did not actually observe them experimentally.

They suggested that the period of the detected oscillations might be the same the period of vibration of the electrons through the grid of the valve, and show that this would result in a relation between wave-length and grid voltage given $\lambda^2 V = \text{constant}$. Most commercial valves will not produce oscillations whose wa length follows this equation when the grid voltage is varied over a wide ran probably because of resonant circuits inherent in the electrode structure.

To look for such oscillations with the present plane-electrode valve, the exter circuit was removed and the potentials were applied through radiofreque chokes connected to the electrode leads immediately outside the envelope. This l 12 cm. of nickel rod with no terminating condenser as the only possible hi frequency circuit external to the electrodes. Its maximum resonant wave-len would be 24 cm. With the cathode-temperature sufficiently high to provide spa charge-limited grid-current throughout, oscillations were maintained with t circuit for all grid-voltages between 35 volts and the highest value tested, cor sponding to a wave-length range of from 220 to 130 cm. The oscillation intensi measured on a loosely coupled crystal circuit, increased continuously with t energy-input to the valve. The curve of wave-length against grid voltage is shown figure 3 and evidently lies close to a curve obeying the Barkhausen relation.

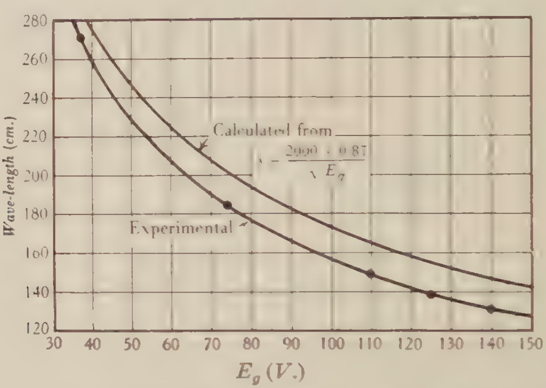

Figure 3. Variation of wave-length with grid voltage for oscillations generated by the plane-electr valve with no external circuit. The cathode-temperature is high enough to supply space-cha limited grid current over the whole range of E_g. Cathode-grid distance, 8·7 mm.; grid-p distance, 10·4 mm.

The agreement between these two curves means that the period of the osc lations is linearly related to some electron transit time over the whole range of t experimental curve. It is found that the period is almost exactly equal to t transit time from cathode to grid and back, space-charge being neglected in t calculation. The oscillation mechanism seems to be confined to the space betwe cathode and grid, and three further facts confirm this view. Firstly, for thr different values of grid-cathode distance the period of the oscillations agreed co sistently with the return transit time across this space. Secondly, variation of t grid-plate distance from 5 to 17 mm. in a typical case does not change the wa length by a detectable amount; although it affects the intensity considerably. Finally, earthing either grid or cathode leads just outside the valve stops the oscillations, while earthing the plate lead has only a very small effect on their intensity.

Reducing the cathode temperature at any fixed grid voltage causes the oscillations to disappear when the grid current falls appreciably below the space-charge-limited value. Their intensity remains constant at its maximum value for all cathode-temperatures above that required to produce this limited grid current.

Applying a positive plate-potential reduces the intensity of the oscillations, and they disappear when the plate is sufficiently positive to collect most of the electrons passing the grid. Thus although the position of the plate does not affect the fundamental mechanism of the oscillations, its presence is necessary to provide a returning stream of electrons in the grid-cathode space.

The theoretical curve of figure 3 is based on an electron transit time derived by neglecting space-charge. Megaw(10) has shown that calculations for a plane-electrode valve similar to those of McPetrie(11) for a cylindrical valve indicate an increase in transit time of just 50 per cent when complete space-charge is present. There is, however, no existing experimental proof of this fact for a positive-grid triode. The agreement of the two curves of figure 3 may prove either that the effect does not exist at all, or merely that it does not apply to the particular vibration in the electron motion which causes these oscillations.

A few commercial cylindrical electrode valves are found to give results similar to those shown in figure 3, with a slight change in experimental procedure. Filament-temperature is found to be a critical variable with these valves, and for each grid voltage there is an optimum value of heating-current. This optimum value is in general that which places the operating point on the upper knee of the grid-current saturation curve. Figure 4 shows a curve of wave-length against grid voltage obtained from an Osram R valve, together with a curve obeying the Barkhausen relation. This valve has a plate-diameter of 10 mm., a grid-diameter of 4·5 mm., and a grid-transparency of about 80 per cent. Circuit conditions are identical with those described for figure 3, chokes being connected in all the leads immediately outside the valve envelope. Heating-current has been adjusted to the optimum value for each experimental point.

Here again it is evident that the transit time of the electrons is the factor controlling the wave-length. Calculation shows that the transit time corresponding to the oscillation period in this case is that from filament to plate and return. This calculation is confirmed by the fact that earthing either plate or cathode terminals stops the oscillations.

Figures 3 and 4 may both be obtained equally well if the chokes are removed from the potential leads, provided these leads are kept well separated, so that no resonant circuit of low damping is formed by them.

The conclusion to be drawn from the results of this section is that the source of the basic oscillations lies in the space charge itself, and that they are entirely independent of and uncontrolled by any external resonant circuits. They are in fact the oscillations which have been referred to throughout the literature Barkhausen-Kurz oscillations in contrast to Gill-Morrell oscillations, and ha been sought by a number of experimenters, usually with negative results. Moore' and Kapzov and Gwosdower(12) have shown some evidence for their existence ov narrow ranges of grid potential.

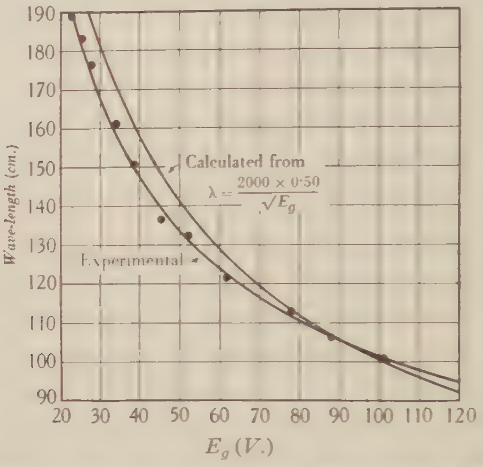

Figure 4. Variation of wave-length with grid voltage for oscillations generated by a cylindric electrode valve with no external circuit. The filament current is adjusted to an optimum va for each grid voltage. Osram R valve. Grid-radius, 2·2 mm. Plate-radius, 5·0 mm.

The production of oscillations by a space-charge-limited current has be discussed theoretically by Benham(14) and Llewellyn(15), but the present resu cannot be regarded essentially as a confirmation of their theories. Benham's the predicts oscillations in a diode, while oscillations could not be obtained from t plane-electrode valve unless electrons were allowed to return through the grid the grid-plate space. Llewellyn's calculations are applied to a positive-grid trio but the complex and varying relation between oscillation-period and electr transit time given by his equations cannot be compared with the simple and ex ratio found in these experiments.

§ 5. OSCILLATIONS WITH EXTERNAL CIRCUITS

When a low-resistance parallel-wire external circuit having suitable resona wave-lengths is connected to any pair of electrodes of the plane-electrode val the experimental results assume several new aspects. It is found that the wav length of all oscillations obtained lies very close to one of the resonant wave-leng of the circuit, and variation of any parameter except circuit-length causes only ve small continuous changes of wave-length. Since the electrodes are alwa approximately at a voltage antinode of the standing waves on the circuit, the reson wave-lengths of a circuit of length L terminated by a low impedance condenser

$4L$, $4L/3$, $4L/5$, etc. It is also found that space-charge-limited grid current is no longer necessary for the production of oscillations. Grid-currents as low as 1 per cent of the limited value may be sufficient.

A typical experimental curve is that shown in figure 5. This is taken with a circuit of fixed length connected to the grid and plate terminals. Grid voltage, grid-current and cathode-grid distance are maintained constant. The curve shows variation of wave-length and plate current (as an approximate measure of oscillation-intensity) with grid-plate distance. The grid current is only 1·5 per cent of the saturation value for the voltage and dimensions involved, so there can be no possible effects of space charge on the electron-motion.

There are seven distinct regions of oscillation in the curve, across each of which the wave-length is constant within 1 cm. The various wave-lengths correspond to the 1st, 2nd, 3rd, 4th and 5th harmonics of the circuit-length—in other words, the standing waves comprise 3, 5, 7, 9 and 11 quarter wave-lengths respectively.

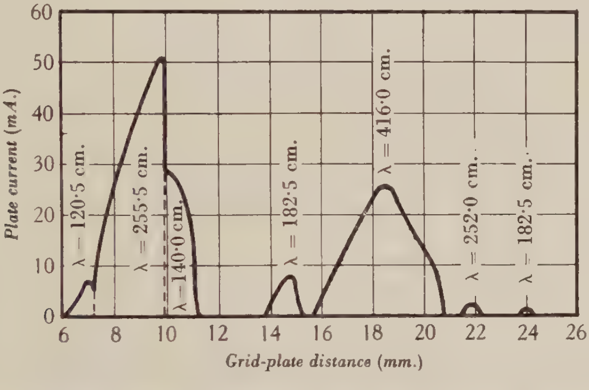

Figure 5. Oscillation regions obtained with the plane-electrode valve connected to an external circuit, on varying the grid-plate distance. All other variables are held constant throughout. External circuit-length from terminating condenser to centre of cathode, 310 cm.; grid potential, 144·2 V.; grid current, 0·75 mA.; cathode-grid distance, 5·5 mm.

The quantity $\lambda\sqrt{E_g}/2000$ expresses, for any oscillation generated in the absence of space charge, the distance across which the return electron transit time for the grid voltage E_g is equal to the period of the oscillations of wave-length λ. It may thus be taken as indicating which pair of electrodes is involved in the generation of any particular region. The value of this product for the seven regions of figure 5 together with the most probable analysis of the electrodes involved is given in table 1.

The analysis shows that the two most important oscillation regions appear to involve a cathode-plate electron-transit, while the three regions second in importance fit a grid-plate electron transit, and the weakest oscillations are those which seem to be related harmonically to the electron-motion. The conception of oscillations whose period is a simple fraction of the electron transit time between two electrodes is contained in every applicable theory, and these dwarf waves have been investigated experimentally at some length by Potapenko$^{(2)}$.

It should be noted that while the harmonic wave-lengths of a parallel-wir circuit having one end closed are proportional to 1, $\frac{1}{3}$, $\frac{1}{5}$, etc., the wave-length corresponding to harmonics of the electron-motion are proportional to 1, $\frac{1}{2}$, $\frac{1}{3}$, etc so that there will be very few occasions where oscillations of harmonically relate frequencies can be simultaneously maintained with a fixed external circuit. Th fundamental wave-length corresponding to the electron motion in the sevent region of figure 5, for example, would be about 550 cm. This is not even approx mately equal to any resonant wave-length of the connected circuit, and henc oscillations of this wave-length could not be maintained.

The position of the plate given in table 1 for each region is that for maximu plate-current. This may introduce a slight error because many experiments hav shown that maximum plate current does not in general correspond to maximu oscillation-intensity. The effect could only be appreciable in the two widest region however, and an inspection of the possible correction shows that the analyses wou not be affected.

Table 1. The grid voltage is constant at 144·2 V., and the cathode-grid distance at 5·5 mm.

Oscillation region	Optimum grid-plate distance (mm.)	Corresponding cathode-plate distance (mm.)	Wave-length λ (cm.)	$\frac{\lambda\sqrt{E_g}}{2000}$ (mm.)	Probable harmonic order	Space in whic oscillations ar probably located
1	7·0	12·5	120·5	7·2	Fundamental	Grid-plate
2	9·8	15·3	255·5	15·4	Fundamental	Cathode-plat
3	<9·9	<15·4	140·0	8·4	Fundamental	Grid-plate
4	14·8	20·3	182·5	10·6	1st Harmonic	Cathode-plat
5	18·7	24·2	416·0	24·9	Fundamental	Cathode-plat
6	21·8	27·3	252·0	15·1	1st Harmonic	Cathode-plat
7	24·1	29·6	182·5	10·5	2nd Harmonic	Cathode-plat

A large number of experimental curves similar to figure 5 have been obtain for different grid-potentials, circuit-lengths, grid-currents, and cathode-grid di tances, and all submit to similar analysis leading to the same general conclusio Figure 6 shows such a curve taken with a total circuit-length about one-tenth of th used for figure 5. The grid-potential and cathode-grid distance are approximat the same as before, but the grid current is about 10 per cent of the space-char saturation value. The grid-plate capacity in this case forms a very appreciable pa of the external circuit, and moving the plate through 20 mm. causes the fundamen wave-length of the net circuit to vary by nearly 30 cm. The slight rise in wav length across the second and third regions will not be discussed here, but it obvious that the mean wave-length of the regions decreases as the grid-pla capacity is decreased. The analysis of this curve is given in table 2. It is essentia the same as that for figure 5. The production of such a small number of regions due to the fact that the short connected circuit has only one resonant wave-leng at which oscillations might be expected to occur. The first harmonic wave-leng of the circuit is 44 cm., and oscillations of this wave-length could be generated at the potential used only as dwarf waves of 4th or higher order.

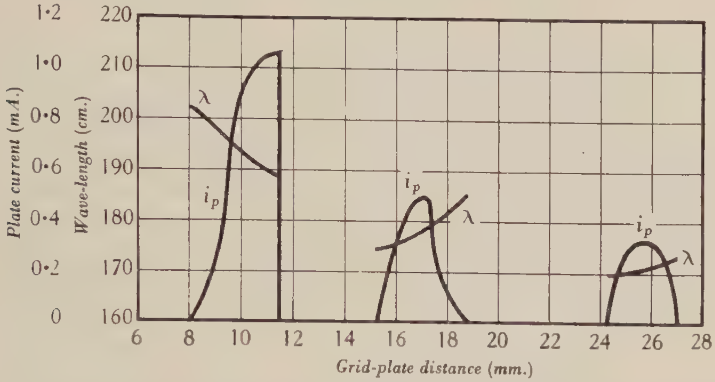

Figure 6. Oscillation regions obtained with the plane-electrode valve connected to a short external circuit, on varying the grid-plate distance. All other variables are held constant throughout. External circuit-length from terminating condenser to centre of cathode, 32·7 cm.; grid voltage, 146·5 V.; grid current, 6·0 mA.; cathode-grid distance, 4·8 mm.

Table 2. The grid potential is constant at 146·5 V. and the cathode-grid distance at 4·8 mm.

Oscillation region	Optimum grid-plate distance (mm.)	Corresponding cathode-plate distance (mm.)	Wave-length λ (cm.)	$\frac{\lambda\sqrt{E_g}}{2000}$ (mm.)	Probable harmonic order	Space in which oscillations are probably located
1	11·5	16·3	188·5	11·4	Fundamental	Grid-plate
2	17·0	21·8	178·5	10·8	1st Harmonic	Cathode-plate
3	25·7	30·5	171·0	10·3	2nd Harmonic	Cathode-plate

§ 6. RELATIONS BETWEEN WAVE-LENGTH AND THE BASIC PARAMETERS

The experiments of § 5 have shown that the wave-length of electron oscillations, where an external circuit is used, depends on three primary parameters. These are circuit-length, inter-electrode distance, and grid potential. It has been assumed that wave-length is related linearly to the first two of these and is inversely proportional to the square root of the grid potential. This assumption has been shown to be justified at a number of isolated values of the parameters.

Very simple experiments serve to show that these relations hold accurately over wide continuous ranges of all three parameters. The curve of figure 7 is obtained by measuring wave-length at several values of circuit-length, with plate-position adjusted for maximum plate current at each circuit-length. The circuit is connected to the grid and plate of the valve. Grid-potential, grid-current, and cathode-grid distance are constant throughout. The curve proves the linearity of the relation between wave-length and circuit-length, and between wave-length and inter-electrode distance, as exactly as the accuracy of experimental measurement wi permit.

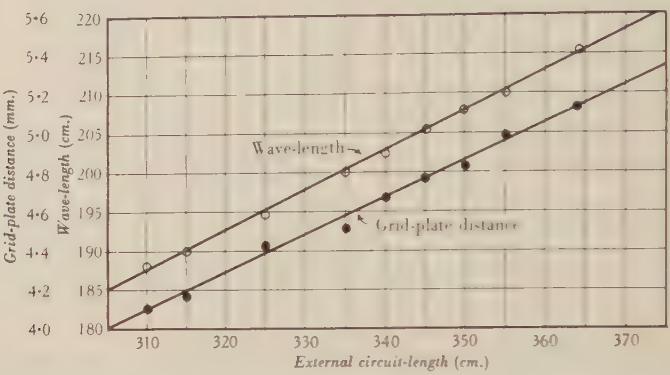

Figure 7. Relation of wave-length and optimum grid-plate distance to external circuit-length fo typical region of oscillations generated by the plane-electrode valve. All other variables he constant throughout. Grid potential, 24 V.; grid current, 3·3 mA.; cathode-grid distanc 5·5 mm.

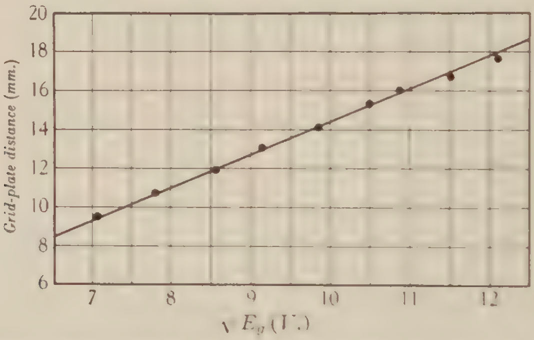

Figure 8. Relation between grid voltage and optimum grid-plate distance at constant wave-leng for oscillations generated under idealized experimental conditions. Constant wave-leng 235 cm.; external circuit length, 170·5 cm.; grid current, 3·0 mA.; cathode-grid distan 8·4 mm.

By keeping the circuit-length and hence the wave-length constant, an int relation between grid potential and electrode spacing for optimum conditions c be obtained, as in figure 8. The cathode-temperature is low enough to prevent t grid current from becoming space-charge-saturated at any of the grid potenti used. The plate-position is adjusted for maximum oscillation-intensity, measur on a crystal circuit, at each point. The assumed relation between the parameters again proved within experimental accuracy.

§7. OSCILLATION MECHANISM AND THEORY

It seems necessary to regard the two types of oscillation described in §§ 4, 5 as produced by distinctly different mechanisms. This hypothesis is not really a new one, but there has not previously existed satisfactory experimental evidence to justify it. The experiments in § 4 prove conclusively that a space-charge-limited current flow in a triode whose grid is at a high positive potential, and whose plate is at cathode potential, is capable of radiating high-frequency energy without the assistance of any resonant inductance-capacity circuit whatever. They also prove that the oscillation period of this radiation corresponds to some electron transit time in the valve, and hence that the oscillations are characteristic of the electron-motion. The radiation is obtained only when the grid current is space-charge-limited. These oscillations may be designated by the name *space-charge electron oscillations*.

The oscillations at very low grid current dealt with in § 5 are produced only when a resonant circuit is connected to the electrodes, and the oscillation wave-length is fixed at some harmonic wave-length of this circuit. The grid current need only be sufficient to supply a certain minimum amount of energy to the circuit, and may be a fraction of 1 per cent of the space-charge-limited value. The function of the electron-motion can only be the production of a negative resistance between the electrodes of the valve, to neutralize the positive resistance of the resonant circuit. These oscillations may be designated by the name *negative-resistance electron oscillations*.

It is not the intention here to develop any mathematical theory for the oscillations. There are so many variables involved that this must inevitably be a very difficult problem. As far as the space-charge oscillations are concerned, some theory analogous to those of Llewellyn and Benham may eventually be developed, but to explain the present experimental results it is necessary to consider two parameters which have not previously been included in space-charge theories. These are the grid-transparency and the distribution in phase of the electrons returning from the plate to the cathode-grid space. Only variation of the latter can account for the change in intensity of space-charge oscillations in the grid-cathode space observed on varying the plate-position.

With regard to the negative-resistance oscillations, Gill and Morrell(16) showed many years ago by very simple mathematical treatment that negative resistance may exist between the grid and plate of a positive-grid triode at frequencies approximately resonant fundamentally or harmonically with the electron transit time between these two electrodes. Alfven(17) has shown by much more thorough analysis involving all possible parameters that negative resistance may also exist between cathode and plate at frequencies similarly related to this longer transit time.

It may therefore be considered that there does exist some theoretical explanation of the oscillation regions of figure 5 and figure 6. The numerical evaluation of the results of Alfven's theory is unfortunately quite impracticable, so that direct comparison of theory with experiment is not possible. No other theory is sufficiently complete to give such comparison any significance. Negative-resistance theories a suggest that maximum oscillation-intensity does not in general occur when th oscillation-period is exactly equal to some electron transit time, but rather that th optimum ratio of these two may vary on either side of unity by several per cent, a a function of a number of the involved variables. It is questionable, however whether indeterminacies in the electron motion can ever be sufficiently eliminate to enable this implication of the theories to be tested experimentally.

§ 8. ANALYSIS OF MORE COMPLEX RESULTS

It is desired to draw only three general conclusions from the experiment results given in this paper. The first of these is the necessity of recognizing tw distinct oscillation mechanisms, as outlined in the last section. The second is tha the period of electron oscillations generated in a triode under idealized experiment conditions is always very nearly equal to the electron transit time between on pair of the three electrodes. The third is that it is possible to obtain oscillation of period corresponding to each of the three possible inter-electrode transit time

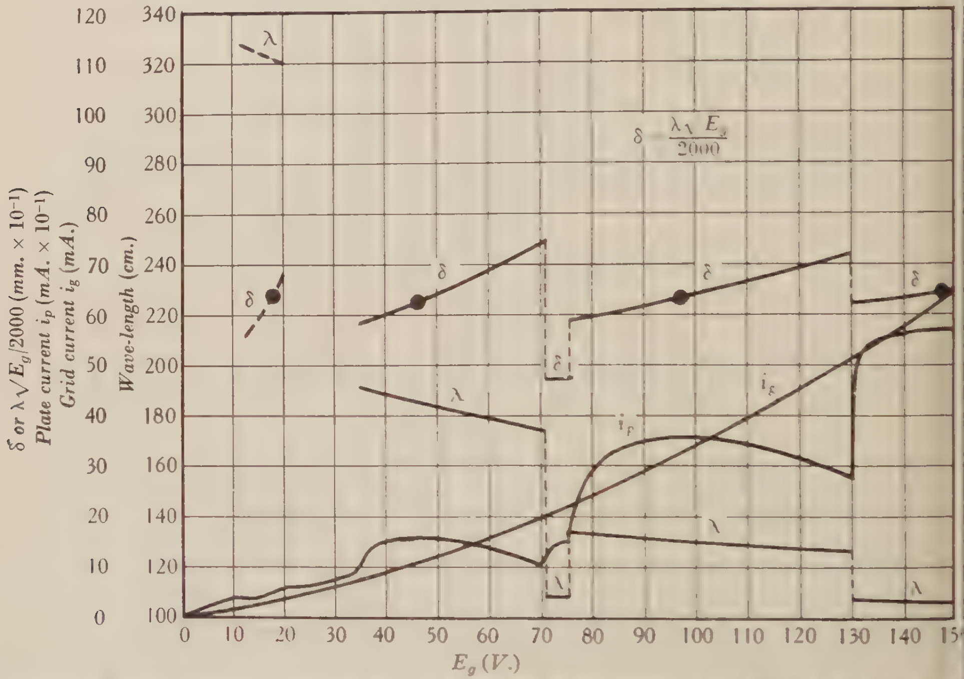

Figure 9. A more complex experimental curve that can be explained on the basis of the conclusion drawn from idealized experiments. All variables except grid potential are kept constant throug out. External circuit-length, 230 cm.; cathode-grid distance, 6·4 mm.; grid-plate distanc 4·9 mm. The black circles mark the value of δ, defined as $\lambda\sqrt{E_g}/2000$, at maximum plat current in each oscillation region. These are the points used for the analysis in table 3.

On the basis of these conclusions drawn from simplified experimental results is possible to analyse much more complex results, from which the same conclusion could not have been initially deduced. Figure 9 is an example of these more comple results. The experimental conditions under which it has been obtained are a combination of those used in §§ 4, 5 above, in that an external circuit is connected to the grid and plate *and* the grid current is space-charge-limited. Electrode-spacings and cathode-temperature are kept constant throughout. The analysis of the five oscillation regions is given in table 3.

Table 3. The cathode-grid distance is constant at 6·4 mm., and the grid-plate distance at 4·9 mm., so that the cathode-plate distance is 11·3 mm.

Oscillation region	Optimum grid potential (V.)	Wave-length λ (cm.)	$\frac{\lambda\sqrt{E_g}}{2000}$ (mm.)	Probable harmonic order	Space in which oscillations are probably located
1	15	325	6·3	Fundamental	Cathode-grid
2	45	185	6·2	Fundamental	Cathode-grid
3	75	108	4·7	Fundamental	Grid-plate
4	97	131	6·4	Fundamental	Cathode-grid
5	142	106	6·3	Fundamental	Cathode-grid

Although the external circuit is connected to grid and plate, the analysis suggests that the oscillations in four of the regions are taking place between grid and cathode. A negative resistance explanation cannot be applied, and it must be concluded that the oscillations in these four regions have their source in the grid-cathode space charge, and are in effect space-charge oscillations, their wave-length being fairly narrowly controlled by the closely coupled low-resistance external circuit. The fifth region seems to be due to negative resistance in the grid-plate space. Since the grid-plate distance is smaller than the grid-cathode distance, and the grid-transparency is only 67 per cent, the total current in the grid-plate space can only be a small fraction of that required for space-charge limitation, possibly 10 to 20 per cent.

This experiment shows that if a circuit having a resonant wave-length λ is coupled in some way to a valve generating space-charge oscillations of the same wave-length λ, currents are induced in the circuit which react on the oscillator, and the oscillation-intensity is in general increased. Analogous effects are obtained with any oscillator and a closely coupled resonant circuit. Oscillations obtained under these conditions may be designated by the name *resonant space-charge oscillations.*

The blotting out of negative-resistance oscillations by the resonant space-charge oscillations in this experiment suggests that the latter are produced more efficiently than the former. This view is supported by the fact that it is found that resonant space-charge oscillations can be produced at much lower energy-inputs to the valve than negative-resistance oscillations can. In an extreme case these resonant space-charge oscillations have been observed at a grid potential of 4·2 V., with a grid current of 0·2 mA., making an input of 0·8 mW. It is important in relation to § 3 that this grid voltage is below the lowest ionization or resonance potential of any gas or vapour that might be considered present.

§ 9. CONCLUSIONS

In this paper the effect of continuous variation has been investigated only f the three primary parameters of grid potential, inter-electrode distance and lengt of external circuit. In addition to the primary parameters there are three min variables which have a more or less critical effect on electron-oscillation mechanis These are grid-transparency, plate potential, and space-charge density. In t present work the effects of the last two have been respectively eliminated by keepi the plate always at cathode potential and by using either maximum or negligib space-charge density. With regard to the negative-resistance oscillations, furth experiment indicates that grid-transparency has not a very critical effect on eith wave-length or oscillation-intensity. The results of figure 6 can be duplicated alm exactly with grids of transparency up to 90 per cent. Space-charge oscillatio however, are considerably affected by this variable. The results shown in figur have been repeated for three different grids of approximately the same transparen 67 per cent, but the oscillations produced with a grid-transparency of 80 per ce for example, are no longer confined to the cathode-grid space. It is hoped in future paper to deal in detail with this effect, and also with the results obtained continuously varying each of the other minor parameters.

How far the results of the present work can be applied to experiments cylindrical-electrode valves is not clear. There seems never to have been a indication that oscillations obtained from commercial valves can be restricted to t grid-filament space or to the grid-plate space as has been found here. The criti effect of heating-current on the space-charge oscillations generated by a comme cial valve, figure 4, also suggests an inherent difference between the oscillati mechanisms for plane and cylindrical electrodes.

§ 10. ACKNOWLEDGMENTS

It is a pleasure to acknowledge my indebtedness to Mr J. A. Ratcliffe for ma helpful suggestions, and to thank him for his continued interest in this work. wish also to thank the McGill University chapter of the Delta Upsilon fratern for a scholarship which enabled this work to be started.

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THE EFFICIENCY OF SEPARATION OF HYDROGE AND DEUTERIUM BY ELECTROLYSIS

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ABSTRACT. A method of determining the efficiency of concentration of D_2O electrolysis of mixtures of H_2O and D_2O is described; it takes account of the effects losses by evaporation and spraying. It is found that the hydrogen-ion concentration the electrolyte and the nature of the electrodes do not affect the efficiency α, but α increase with increasing current-density. The coefficient α is found to be $4{\cdot}0 \pm 0{\cdot}2$ with curren density 0·6 A. cm^{-2}, and $4{\cdot}6 \pm 0{\cdot}1$ with current-density 2 A. cm^{-2} These results are agreement with values predicted theoretically by Urey and Teal.

§ 1. INTRODUCTION

EXPERIMENTAL work hitherto published shows a marked uncertainty n only of the value of the coefficient of separation of hydrogen and deuteriu by the electrolysis of water under given conditions, but also of the effect o the coefficient of changes in these conditions; thus, published values vary betwee 2·6 and 7·3. Bell and Wolfenden(1) found that the separation coefficient α depen solely on current-density, being greatest for high current-densities, while Collie maintains that for efficiency a current-density less than 0·1 A. cm^{-2} is necessar On the other hand, Topley and Eyring(3) found a dependence of α upon the natu of the electrodes and upon the hydrogen-ion concentration of the electroly Collie(2) also obtained different values of α with different experimental condition

Values of α obtained by other workers are given in numerical order in table together with the nature of the electrodes and electrolyte, the current-density an the mean D_2O concentration. A study of this table fails to indicate any certa dependence of α on any of the above factors, but it should be borne in mind that some cases experimental errors are admitted to be large, and apparently no co rection has been made for the inevitable loss of water by evaporation and sprayi during electrolysis.

In this paper is described an attempt to measure α with greater certainty ar to study the effects on α of four factors, the current-density, the nature of t electrodes, the hydrogen-ion concentration (pH) of the electrolyte and D_2O co centration.

§ 2. THEORY OF THE EXPERIMENT

When a mixture of D_2O and H_2O is electrolysed, the proportion of D_2 molecul to H_2 in the evolved hydrogen gas is less than the ratio of D_2O to H_2O in the electr

Table 1

Reference number	Electrolyte	Electrodes	Current-density A./cm.²	Concentration of D_2O	α
2	H_2SO_4	Pb	0·1	0·002	2·6
1	NaOH	Ni	0·07	0·001	3·7
1	NaOH	Ni	—	0·004	4·0
1	NaOH	Ni	—	0·002	4·8
13	NaOH	Ni	—	—	5·0
3	KOH	Ag	1	0·07	5·2
1	NaOH	Pt	—	0·002	5·3
1	NaOH	Cu	—	0·002	5·3
3	KOH	Ni	1	0·07	5·5
3	H_2SO_4	Cu	1	0·07	5·6
1	NaOH	Ni	10	0·001	5·6
3	H_2SO_4	Pt	1	0·07	5·7
4	NaOH	Ni	0·7	—	5·8
3	KOH	Cu	1	0·07	6·8
3	KOH	Pt	1	0·07	7·0
3	KOH	Fe	1	0·07	7·2
3	KOH	Pb	1	0·07	7·3

te. A separation coefficient α may be defined as follows. Let a small fraction f the water be decomposed electrolytically; then the fraction of H_2O molecules ecomposed will be α times that of D_2O molecules. α

The relation between α and the initial and final volumes and D_2O-concenrations of the water has been given by Harteck⁽⁴⁾, but losses by spraying and vaporation of the solution during electrolysis modify the theoretical equation which e gives as is shown below.

Let N_0 be the number of molecules of H_2O in water initially; N the number of nolecules of H_2O in water at time t; n_0 the number of molecules of D_2O in water nitially; n the number of molecules of D_2O in water at time t; K the total number f molecules electrolysed per second; and k the total number of molecules evaporated er second. N_0, N n_0 n, t, K k

Now the vapour pressure of D_2O is about 0·9 times that of H_2O⁽⁵⁾, but for a orrection for loss by evaporation the assumption that the vapour pressures are qual gives a sufficiently close approximation. Now for a small loss by electrolysis,

$$\frac{(dN)_1}{(dn)_1} = \frac{N}{n}\alpha \qquad \text{......(1·11)},$$

nd for a small loss by evaporation,

$$\frac{(dN)_2}{(dn)_2} = \frac{N}{n} \qquad \text{......(1·12)},$$

vhere the suffixes 1 and 2 imply electrolysis and evaporation respectively. Further, he number of molecules lost in time dt is given by 1, 2

$$(dN)_1 + (dn)_1 = -K\,dt \qquad \text{......(1·21)},$$

nd

$$(dN)_2 + (dn)_2 = -k\,dt \qquad \text{......(1·22)}.$$

From equations (1·11) and (1·21),

$$(dN)_1=-\frac{KN\alpha}{N\alpha+n}\,dt \qquad \text{......(1·31)},$$

$$(dn)_1=-\frac{Kn}{N\alpha+n}\,dt \qquad \text{......(1·32)},$$

and from equations (1·12) and (1·22),

$$(dN)_2=-\frac{kN}{N+n}\,dt \qquad \text{......(1·41)},$$

$$(dn)_2=-\frac{kn}{N+n}\,dt \qquad \text{......(1·42)}.$$

Thus the total number of molecules lost in time dt is

for H_2O, $$dN=(dN)_1+(dN)_2=-\frac{KN\alpha}{N\alpha+n}\,dt-\frac{kN}{N+n}\,dt \qquad \text{......(1·51)},$$

and for D_2O, $$dn=(dn)_1+(dn)_2=-\frac{Kn}{N\alpha+n}\,dt-\frac{kn}{N+n}\,dt \qquad \text{......(1·52)}.$$

These equations (1·51) and (1·52) may be combined and integrated to give relation

$$\frac{\alpha-1}{1+r}\log\frac{N+n}{N_0+n_0}=\log\frac{1-C}{1-C_0}-\alpha\log\frac{C}{C_0} \qquad \text{......(1·6)},$$

r, C where $r=k/K$ and $C=n/(N+n)=$ the concentration of D_2O.

This relation is similar to that of Harteck but has the added factor $1/(1+r)$ a the ratio of the initial and final numbers of molecules $(N+n)/(N_0+n_0)$ is su stituted for the ratio of the initial and final volumes.

The ratio $(N+n)/(N_0+n_0)$ is connected with the ratio of initial and final mass

W_0, W W_0, W of water by the relation

$$\frac{W}{W_0}\doteqdot\frac{N+n}{N_0+n_0}\left\{1+\frac{C-C_0}{9}\right\}$$

C_0 very nearly when n/N_0 is small, where C_0 is the initial concentration of D_2O.

§ 3. EXPERIMENTAL PROCEDURE

Conditions for accuracy of α. By differentiating equation (1·6) a relation betwe the errors in W/W_0 and C/C_0 and the resulting error in α may be found. The err in r may be neglected, and also the first term on the right-hand side of the equatio We obtain

$$\frac{d\alpha}{\alpha}=\left[\frac{1}{\log W/W_0}-\frac{1}{\log W/W_0+(1+r)\log C/C_0}\right]\frac{d\,(W/W_0)}{W/W_0}$$
$$-\left[\frac{1}{\dfrac{\log W/W_0}{1+r}+\log C/C_0}\right]\frac{d\,(C/C_0)}{C/C_0} \qquad \text{......(2·1)},$$

$$=1\cdot 6\,\frac{d\,(W/W_0)}{W/W_0}-2\cdot 2\,\frac{d\,(C/C_0)}{C/C_0} \qquad \text{......(2·2)},$$

after substitution of the experimental values given below.

Thus 1-per-cent errors in W/W_0 or in C/C_0 give considerably larger errors in α.

Conditions of experiment. The influence of four factors, (i) current-density, i) the hydrogen-ion concentration of the electrolyte, (iii) the nature of electrodes, d (iv) the concentration of D_2O, has been studied, one of these factors being varied a time.

The experiments were carried out in water-cooled test tubes with either wire r metal-strip electrodes. In some the evaporation losses were found by calculation om the initial and final volumes of water and the input of electrical energy. In e more accurate experiments, however, the latter quantity was not determined, ut losses were measured directly by passing the electrolytic gases through a glass essel cooled with solid carbon dioxide, and weighing the condensed liquid. The ncentration of D_2O was measured by a float method described in a later section.

In a typical experiment the initial concentration of the electrolyte, sodium ydroxide, was 1·4 per cent. The concentration of D_2O was from 2·5 to 10 per cent. he current-density was high—about 2 A./cm.² The electrodes were of nickel wire. he mass of electrolyte was 1·895 g. The masses W_0, W were 133·695 g. and 9·769 g. respectively. Hence it was calculated that the mass of water evaporated as 7·407 g. and the mass of water electrolysed was 106·519 g. The concentration f D_2O in the tap water was 0·0001 per cent; the initial concentration C_0 was ·0246 ± 0·00002, and the final concentration C was 0·0970 ± 0·0003. The corresponding equilibrium temperatures of the float were 17·21 ± 0·005, 26·635 ± 0·005, nd 49·5 ± 0·1° C. Hence from equation (6) α was found to be 4·24 ± 0·05.

§ 4. EXPERIMENTAL RESULTS

Hydrogen-ion concentration of electrolyte. The current-density was about ·2 A./cm.² Electrodes of lead were used with acid solutions, and of stainless steel ith alkaline solutions. The mean concentration of D_2O was 0·002.

Table 2

Number of experiments	Electrolyte	α
3	H_2SO_4	2·9 ± 0·3
13	NaOH	3·1 ± 0·3

Thus the hydrogen-ion concentration of the electrolyte does not appear to ffect α.

Nature of electrodes. The current-density was about 0·6 A./cm.² The electrolyte as sodium hydroxide, from 1 to 6 per cent. The mean concentration of D_2O as 0·002.

Table 3

Number of experiments	Electrodes	α
1	Monel	3·98 ± 0·2
3	Nickel	3·93 ± 0·2

α is independent of the electrodes, as is also shown by the experiment at lower current-density with lead and stainless-steel electrodes given in table 2.

Concentration of D_2O. The current-density was about 0·2 A. cm^{2} The electrod were stainless steel or iron. The electrolyte was sodium hydroxide.

Table 4

Number of experiments	Concentration of D_2O	α
17	0·0015	3·1 ± 0·3
11	0·0045	2·9 ± 0·3

α is a constant for different stages in the process of concentration.

Current-density. The electrodes were of nickel, iron, stainless steel and nick chromium, α being independent of these. The electrolyte was sodium hydroxi The mean concentration of D_2O was 0·002 to 0·06.

Table 5

Number of experiments	Current-density (A./cm^2)	α
22	Low, about 0·2	3·0 ± 0·3
3	Medium, about 0·6	4·0 ± 0·2
3	High, about 2	4·6 ± 0·05

It is evident that the current-density has a marked influence on α, high curre densities being the most efficient. The determination of the actual value of t current-density was rendered very uncertain by such factors as varying depth the electrolyte, current-variations, the shape of the coiled-wire electrodes, and t uncertain conducting surface. The electrodes employed were as follows. F low current-densities, metal strips 2 cm. wide, in 1-in. test tubes; for mediu current-densities, metal strips 1 cm. wide, in ½-in. test tubes; and for high curre densities, metal wires in concentric coils.

Value of α. The eight most accurate results for α are shown in table 6.

Table 6

Current-density (A./cm^2)	0·6	2
α	3·98 ± 0·2 4·30 ± 0·2 3·65 ± 0·2 3·85 ± 0·2	4·84 ± 0·2 5·12 ± 0·08 4·24 ± 0·05 4·48 ± 0·08
	Mean 4·0 ± 0·2	Mean 4·6 ± 0·1

The differences between these readings exceed the experimental errors and due to the variations of current-density, experiments having been performed different test-tubes with different coiled-wire electrodes.

§ 5. MEASUREMENT OF CONCENTRATION OF D_2O

The molecular concentration of D_2O in the D_2O-H_2O mixtures of the above xperiments was determined with a cylindrical pyrex float 1·5 cm. long and 0·5 cm. diameter. This was so constructed as to float in ordinary water at 17·21° C. The ifference between the temperature at which the float neither rose nor sank when ompletely immersed in ordinary water, and a similar temperature for the mixture nder observation, gave a measure of the difference of D_2O concentrations.

The concentration of D_2O in tap water was taken as 1 in 10,000(6), which figure grees with a determination of 1 in 8500 made in this laboratory by Christiansen, rabtree and Laby, but not with earlier estimates(7).

Water samples were distilled five times, a tin condenser being used for all but e first distillation. A small correction was necessary for the thermal expansion the float, but calculation showed that the effect of changes in the volume of the oat with variations of barometric pressure was negligible.

The water sample was placed in a narrow test tube containing the float, and e whole was immersed with a Beckmann thermometer in a large well-stirred ater bath, the temperature of which was varied slowly. Small movements of the oat were easily followed by projecting a magnified image of the apparatus on to a illimetre scale at a distance of about 6 ft.

The calculation of C was carried out by means of a formula derived as follows. aking for the present ordinary distilled tap water as pure H_2O, let C' be the xcess molecular concentration of D_2O in the mixture over that in the tap water, K_1 a constant, d_D the density of pure D_2O, and d_H that of H_2O.

Since the molecular weight of $D_2O=20$ and of $H_2O=18$, the total mass of ater is

$$K_1\,(20n+18N) \qquad \text{......(3·11)},$$

nd the total volume of water is

$$K_1\left(\frac{20n}{d_D}+\frac{18N}{d_H}\right) \qquad \text{......(3·12)},$$

ssuming that when H_2O and D_2O in the liquid state are mixed the volume of the ixture is the sum of the volumes of the constituents.

Hence the mean density

$$\rho=\left(\frac{20n+18N}{20nd_H+18Nd_D}\right)d_H d_D \qquad \text{......(3·21)},$$

$$=\left(\frac{n+0{\cdot}9N}{nd_H+0{\cdot}9Nd_D}\right)d_H d_D \qquad \text{......(3·22)}.$$

But $$C'=n/(N+n) \qquad \text{......(3·31)}.$$

Therefore $$n=C'N/(1-C') \qquad \text{......(3·32)}.$$

Thus from equations (3·22) and (3·32)

$$\rho=\frac{\left(\frac{C'}{1-C'}+0{\cdot}9\right)d_H d_D}{\left(\frac{C'}{1-C'}\right)d_H+0{\cdot}9\,d_D} \qquad \text{......(3·41)}.$$

Whence $$C'\left[10\left\{\frac{1}{d_{11}}-\frac{1}{d_{1}}\right\}-\left\{\frac{1}{d_{11}}-\frac{1}{\rho}\right\}\right]=9\left\{\frac{1}{d_{11}}-\frac{1}{\rho}\right\} \qquad \text{......(3·42).}$$

But ρ is the density of the float at equilibrium temperature t in the mixture. Th ρ' $\rho=\rho'\{1-\beta(t-t')\}$, where ρ' is the density of the tap water at equilibrium ter t', β perature and t' that of the float in tap water, and β is the volume coefficient of therm expansion of the float.

Finally C, the true concentration of $D_2O=C'-0{\cdot}0001$, since tap water h already one part in 10,000 of D_2O.

C could thus be calculated since ρ, d_{11}, d_{1}, were known and β ($3{\cdot}1\times10^{-6}$) w measured by determining the resting-point of the float in a solution of sodiu chloride of known density.

§ 6. CONCLUSIONS

The experimental results given indicate that α, the efficiency of concentratio depends solely on the current-density, a conclusion which is in agreement wi those of Bell and Wolfenden⁽¹⁾, but not with those of Collie⁽²⁾ or Topley an Eyring⁽³⁾.

The actual value, 4·6, of α obtained with high current-density is lower than t mean, 5·4, of values given in table 1, but is in better agreement with the theoretica predicted values.

The coefficient α varies with temperature⁽⁸⁾, but results given in this pap and in table 1 all relate to room-temperature.

The theoretical deduction of the value of α is complicated by the fact th equilibrium conditions do not hold in practice during electrolysis, and by t necessity of making various simplifying assumptions. The equilibrium consta of the reaction

$$H_2+HDO=HD+H_2O$$

leads⁽⁹⁾ to a value of α equal to 3 and equilibrium conditions have been obtain approximately by Washburn, Smith and Frandsen⁽¹⁰⁾, who found values of α 1·49, 2·71, 3·14, 2·81 and 2·79.

α has been derived theoretically by Bell⁽⁸⁾ and by Topley and Eyring⁽¹¹⁾, w gave a value of about 20; their theory has been criticized by Urey and Teal⁽¹²⁾, w calculate values 4·0, 4·3 and 4·55 at 0, 50 and 100 moles per cent of D_2O respective These are in better accord with experiment.

§ 7. ACKNOWLEDGMENTS

The writer has been in receipt of a research grant from the University Melbourne while he was engaged on this investigation, the expenses of which we provided by a gift to this University from the Associated Smelters Company Australia. Mr Germann, of the Melbourne and Metropolitan Tramways Boa very generously made available the electrolyte contained in alkaline batteries whi had become enriched in deuterium. The writer wishes to thank Prof. T. H. Lal F.R.S., for suggesting the investigation and for his valuable help throughout.

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A TRICHROMATIC COLORIMETER

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ABSTRACT. A trichromatic colorimeter is described in which the integrating properti of a diffusing sphere are employed to effect the mixture of the three instrumental stimu Means are described for obtaining linearity of the scales. The general construction h been planned with a view to keeping the design as simple as is consistent wi efficiency.

§ 1. INTRODUCTION

ROUTINE practice of colour-measurement for scientific and industrial pu poses makes considerable demands on the measuring-instrument in the w of such requirements as accuracy, simplicity of construction, ease of mai tenance. At present these demands seem to be most adequately met by the Gui trichromatic colorimeter(1) manufactured by Messrs Adam Hilger Ltd. For vario reasons it is thought the production of another colorimeter of equal accuracy an efficiency would be of value. The Guild colorimeter is an expensive instrumen The original model was designed and made for research work at the Natio Physical Laboratory, where special supplies of direct current can be obtained required. There is some difficulty, however, in adapting it for use with alternati current, on account of the method of colour-mixing employed. In view of t progressive change from d.c. to a.c. in the public supply throughout the count this disadvantage is becoming of increasing importance. The present instrum can be used equally well with d.-c. or a.-c. supply. The cost of construction shou be considerably less than that of the Guild instrument, and it should theref find a field of utility wider than that which was available when the Guild colo meter alone could be considered an instrument of high accuracy.

The general principle underlying the design of visual trichromatic colorimet may be briefly summarized as follows. Provision is made for the coloured li from the specimen under examination to fill one half of a bipartite photomet prism. The other half is filled by a mixture of the three matching-stimuli. Th matching-stimuli or working primaries are colours of high saturation, usuall red, a green and a blue. Means are also provided to vary independently and i known manner the intensities of the matching-stimuli. The basis of operat consists in finding the intensities of these stimuli which give a colour-match w the specimen.

As attention here is confined to the construction of the instrument only, for particulars regarding method of use, transformation of results to standard form, etc. reference should be made to the paper quoted above(1) and also to the paper on the C.I.E. colorimetric system by Smith and Guild(2). The trichromatic colorimeters which have been described from time to time differ only in the methods by which the matching-stimuli are produced, mixed, and varied in intensity. In the Guild instrument coloured gelatine filters illuminated by a gas-filled lamp constitute the matching-stimuli. The mixing of these is effected by a rotating prism which transmits each stimulus successively to the field of view with sufficient rapidity for no flicker, either of colour or of brightness, to be observed. The method of variation of intensities can be regarded as an application of the principle of the variable rotating sector, but instead of the customary combination of a rotating sector and a stationary beam of light, a prism which rotates the beam of light and a stationary sector are used. In the instrument now to be described the chief feature of the design is the replacement of the rotating prism by a diffusing integrating sphere.

§ 2. CONSTRUCTION OF THE INSTRUMENT

Figure 1 shows the plan of the instrument and figure 1*a* an end elevation showing the plate *A* which faces the light-source.

The light-source *L* is a 250-W. lamp of the projector type run at 90 per cent of its rated volts. This reduction of voltage ensures longer life and greater uniformity in emission of radiation. A source of this rating at least is needed if a sufficiently high field-illumination is to be obtained.

The three rectangular apertures *R*, *G*, *B* in the plate *A* can be uncovered to varying extents by shutters sliding in V-shaped grooves. The dimensions of each aperture are 8·5 cm. × 2·0 cm. Each shutter is operated by a separate rack and pinion. A scale mounted on the shutter allows the extent of the opening to be read. The edges of the shutter and aperture are bevelled, and special attention has been paid to the fit when the shutter is on the point of closing.

Behind the apertures are mounted the colour filters, the red being placed in aperture *R* and so on. For reasons of permanence the red and blue filters are of glass. The red is a selenium red and the blue is Chance's No. 7 Contrast filter. As no suitable green glass seems to be available at present, the green Wratten filter as used in the Guild instrument has been retained.

The condenser lens *C* which follows is of the usual type. It was manufactured from plate glass of good quality; its diameter was 6 in. and the radius of curvature of the curved surfaces was also 6 in. The lens collects the light transmitted by the three apertures and brings it to a focus at the opening *D* in the sphere *S*.

The sphere is 6 in. in diameter, is made of copper, and is silver-plated inside. A layer of magnesium oxide is smoked on to the silver in the usual way. The apertures *D* and *F* are 1·3 in. and 0·25 in. respectively in diameter. Glass windows covering these apertures prevent the magnesium oxide from being contaminated with dust. The appropriate half of the photometric prism P is illuminated by light from the sphere-wall at E via opening F and prism H. A silvered mirror of thi glass can be substituted for the prism H. The function of lens O is to produce a

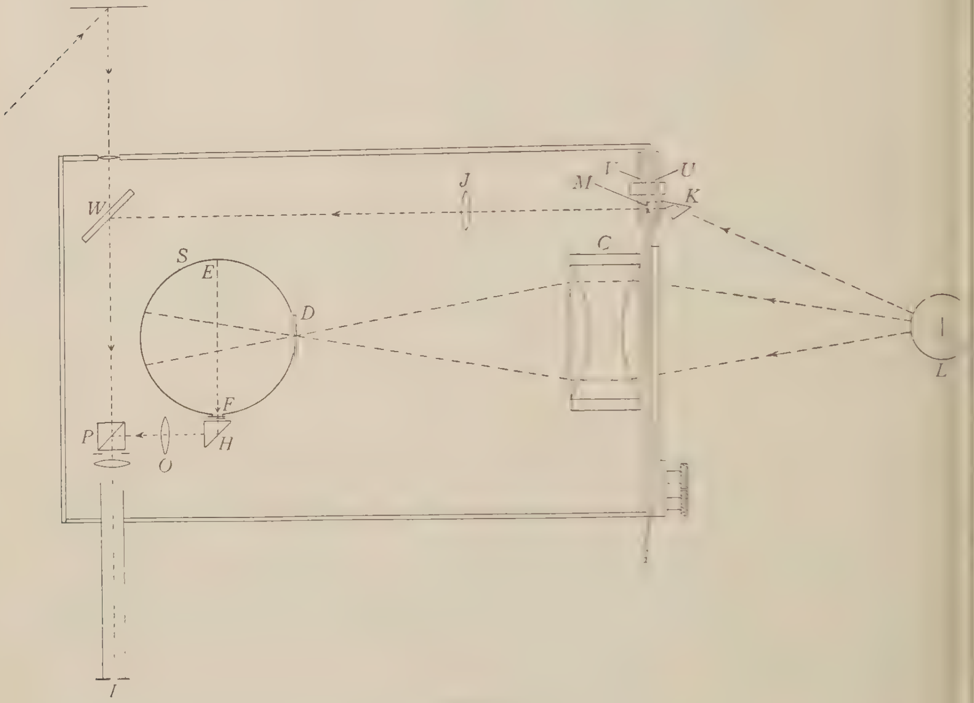

Figure 1.

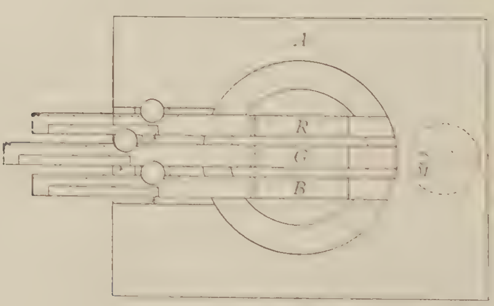

Figure 1 *a*.

image of the observation pupil I at F in order that the opening F may be kep small. As the sphere by repeated reflections integrates the light which enters at I the light observed in the photometric prism is a mixture of the light transmitte by the three apertures, and the intensity of each component is approximatel proportional to the scale-reading on the shutter.

In other respects, such as the method of obtaining the standard size of field and the optical system for viewing the specimen, the construction follows that employed in the Guild instrument. There is a slight variation in the arrangement for adding desaturating colour to the colour under test. The prism K acts as a mirror and deflects the light from L through M. In the small aperture there is mounted a piece of glass ground on both sides to act as a diffuser. The remainder of the system, comprising the plate U to hold the colour filters, the circular photometer wedge V, the lens J, and the glass plate W is similar to the corresponding part of the Guild instrument.

§ 3. LINEARITY OF THE SCALES

For a colorimeter to give a satisfactory performance the standard of accuracy required in the scales of the matching stimuli is such that, when calibrated against sectors by means of brightness-matches in the small photometric field, no measurable error should be found. A more sensitive test can be made by comparing the effectiveness of different small parts of the aperture with each other. Experience shows that variations up to 2 per cent from the mean value can be tolerated in this test. In the present design the factors which tend to destroy the proportionality between scale-reading and intensity of stimulus are as follows. (*a*) Uneven horizontal distribution of luminous flux within the cone subtended by the aperture at the source. (*b*) Uneven distribution of light over each aperture due to (i) varying distance from the lamp, (ii) varying obliquity. (*c*) Non-uniform transmission through the coloured filters. (*d*) Varying reflection and absorption losses in the condenser lens. (*e*) Varying effectiveness of the sphere as an integrator for pencils of light entering at different angles.

It should be noted that, on account of the fact that the shutters slide horizontally, vertical variations in the above quantities are of no consequence and only horizontal variations need be taken into account.

The magnitude of (*a*) was measured in the following manner. The projector lamp was mounted on a photometric bench and rotated through the required angle. A photometer head some distance away served as an indicator for variation in brightness. These are the conditions appropriate to the middle aperture. The upper and lower apertures require the projector lamp to be tilted during the measurements. In all cases no measurable variation could be detected. It is important that no obscuring of one filament by another should take place in the cone defined by the aperture. For this reason it is desirable to employ the type of lamp in which the filaments are all in one plane. The effect of (*b*) can be calculated from the inverse-square law together with the cosine law. For the middle aperture the variation amounts to 6 per cent.

To insist on absolute uniformity in (*c*) would involve the rejection of an excessive number of filters. Moreover, to draw up a calibration curve of the variation in transmission which is applicable when the filter is mounted in the instrument is rather difficult. The most practicable course is therefore to reserve the available

2-per-cent variation for this item alone, that is, to permit variations not exceeding 2 per cent in the local transmission through filters. Even with this latitude some difficulty has been experienced in finding a suitable piece of selenium glass for the red filter. To calculate (*d*), rays corresponding to certain points in the apertures were traced through the condenser lens, and the reflection losses at the surfaces calculated from Fresnel's formula and the thicknesses of glass traversed measured. To estimate the absorption losses the absorption per centimetre of the glass for light of the composition transmitted through the apertures was required. This was supplied by separate measurements on a plane slab of the same material as that from which the lens was manufactured. The variation due to these two sources combined was a maximum for the red aperture where it attained a magnitude of 7 per cent.

The following simple test gave a measure of the variation due to (*e*). A bean of light was allowed to enter the sphere and the brightness of a fixed point of the interior wall was observed when the sphere was rotated through an angle equa to the angle subtended at the opening of the sphere by an aperture containing colour filter. No measurable alteration in brightness could be detected. Deterioration of the magnesium-oxide coating can only affect the performance of the instrument by destroying the integrating power of the sphere to such an extent that the above test would produce a positive result. As the interior is completely protected experience with similar surfaces shows that no significant change is likely to take place in a shorter period than three months and the coating therefore need only be renewed at that interval.

By combining (*b*) and (*d*) a calibration curve was drawn for each aperture. I was found that the curve which had been calculated to refer to a line along the middle of the aperture was sensibly identical with those referring to the top an bottom portions and so could be applied to the whole aperture. As effects (*b* and (*d*) work in opposite directions the final correction was much less than had bee anticipated. In the case of the blue there was practically complete compensation with the green the centre of the aperture was 2½ per cent more effective than the ends; with the red the ends were more effective than the centre by 1½ per cent As the application of calibration corrections is undesirable when it can be avoided the need for corrections has been eliminated by modifying the widths of the apertures by the insertion of templates, which compensate for these variations an so render the scale strictly linear. Figure 2 shows these templates exaggerated i size. It is advisable to fit them at both top and bottom so that each shares half o the correction because in this way any errors due to an irregular vertical distribution of light from the lamp are averaged out.

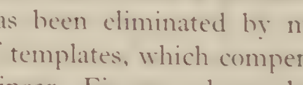

Figure 2. Rectangular aperture modified by templates.

It should be noted that on account of the arrangement of the spiral filaments in a projector lamp the vertical variations in brightness are considerably larger than the horizontal. This fact caused the abandonment of a design in which it was thought the need for calibration corrections would be eliminated by giving the apertures the form of sectorial openings.

A final test of the scales in the completed instrument, made by means of sectors, showed that they were sufficiently accurate for colorimetric measurements. Generally the departures from linearity were less than 1 per cent. In the case of very small openings in which readings cannot be made to this degree of accuracy, the scales were found to be true to the limit to which they could be read.

§ 4. PERFORMANCE

Throughout the design the aim has been to keep the construction simple so far as this could be done without loss of efficiency and, as far as possible, to employ only components which can be easily and cheaply obtained. The only high-grade optical part is the photometric prism. As no correction for chromatic or other aberrations is necessary, the small lenses are ordinary spectacle glasses. Of the metal parts only the apertures and shutters require high-grade workmanship.

The standard of performance of this colorimeter is equal to that of the Guild instrument.

§ 5. ACKNOWLEDGMENTS

In conclusion I should like to thank Mr J. Guild for his interest in the instrument and for the helpful suggestions he has made, and Mr Buxton for his advice on points in the mechanical design. The instrument was constructed in the Optics Workshop, National Physical Laboratory.

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THE INFLUENCE OF A MAGNETIC FIELD ON THE HIGH-FREQUENCY CONDUCTIVITY OF AN IONIZED MEDIUM

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ABSTRACT. The relation between the transverse high-frequency conductivity and th pressure of ionized air under the influence of an imposed magnetic field has been studie experimentally. The pressure at which such conductivity is a maximum is found to var with the intensity of the magnetic field as is to be expected on theoretical grounds the high-frequency conductivity is due solely to electrons. The significance of the resul in connexion with ionospheric conductivity is discussed.

§ 1. INTRODUCTORY

In a previous communication[(1)] from this laboratory an account was given of a experimental investigation of the variation of the radio-frequency conductivit of ionized air with pressure. It was there shown that, from the value of th critical pressure at which such conductivity for a given ionization-content w found to be a maximum, the magnitude of the collisional frictional forces experienc by vibrating electrons could be estimated.

The present communication deals with an extension of such experiments t cases in which a magnetic field is imposed on the ionized medium. This extensio was prompted by the need for checking certain deductions from the magneto-ion theory[(2)] of wireless transmission concerning the absorptive effect of uppe atmospheric conductivity on waves travelling in the ionosphere.

§ 2. THE RADIO-FREQUENCY CONDUCTIVITY OF AN IONIZED MEDIUM IN A MAGNETIC FIELD*

The theory of the propagation of electromagnetic waves through an ioniz medium under the influence of an imposed magnetic field was given by H. Lorentz as early as 1909[(3)]. From the results obtained by Lorentz we can deduc with but slight modifications, the expression which we use here for the radi frequency conductivity of the medium.

We consider the medium as consisting of N electrons per cm.^{3}, each of charge e and mass m, the number of electrons being small compared with the number of uncharged molecules in a given volume of the gas. The conductivity σ with which we are concerned is the transverse value measured for radio-frequency electric forces of angular frequency p, applied at right angles to an imposed magnetic field H_0.

N, e
m
σ
p
H_0

The symbols retaining the same significance, it is easy to show, from a comparison of the equations of Lorentz* and the equations relating to the propagation of electromagnetic waves through a conducting dielectric, that we can regard the ionized medium as possessing a conductivity, transverse to the magnetic field, given by

$$\sigma = \frac{p}{4\pi} \frac{\beta(\alpha^2+\beta^2+\gamma^2)}{(\alpha^2-\beta^2-\gamma^2)^2+4\alpha^2\beta^2} \qquad \text{......(1)},$$

where

$$\alpha = -\frac{mp^2}{4\pi Ne^2} \qquad \text{......(2)}, \qquad \alpha$$

$$\beta = \frac{mp\nu}{4\pi Ne^2} \qquad \text{......(3)}, \qquad \beta$$

$$\gamma = \frac{pH_0}{4\pi Nec} = \frac{mpp_H}{4\pi Ne^2} \qquad \text{......(4)}, \qquad \gamma$$

where

$$p_H = \frac{H_0 e}{mc} \qquad \text{......(5)}, \qquad p_H$$

and ν is the frequency of collisions of an electron with the surrounding neutral molecules, while c is the velocity of light.

ν
c

We have here assumed, following Appleton and Chapman$^{(4)}$, that the frictional force per unit velocity, suffered by a vibrating electron, is equal to $m\nu$. We have also, following Darwin$^{(5)}$, omitted the usual Lorentz polarization term.

(*a*) *Case of zero magnetic field.* If there is no imposed magnetic field, equation (1) reduces to

$$\sigma = \frac{p}{4\pi} \cdot \frac{\beta}{\alpha^2+\beta^2} \qquad \text{......(6)},$$

or, on substitution from equations (2) and (3) for α and β

$$\sigma = \frac{Ne^2}{m} \frac{\nu}{\nu^2+p^2} \qquad \text{......(7)}.$$

This expression reaches a maximum for constant N and p, when $\nu = p$. If, therefore, we find the pressure at which σ is a maximum for constant N and p, we can find ν for that pressure. This is the basis of the method of measuring ν used by Appleton and Chapman.

In discussions of the absorption suffered by radio waves in travelling through the ionosphere it has been usual to assume that since, for a given wave-frequency, the attenuation factor is proportional to σ, the absorption suffered by the waves for given N is a maximum at an atmospheric level for which $\nu=p$. There is no abundant evidence, however, that the propagation of wireless waves is ver materially influenced by the earth's magnetic field, since the effective electric carriers are known to be of electronic mass. It is, therefore, important to fin whether this level of maximum absorption is altered by the presence of such field.

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* Lorentz, *Theory of Electrons*, equation (239), p. 160. In deriving equation (1) we have neglected the additional electric force which Lorentz includes as arising from the polarization at right angles to the applied electric forces. The significance of this omission is discussed later.

(*b*) *Case of imposed magnetic field.* Equation (1) for the high-frequency conductivity transverse to the magnetic field may be written

$$\sigma=\frac{p}{4\pi}\frac{\beta(\alpha^2+\beta^2+\gamma^2)}{(\alpha^2+\beta^2+\gamma^2)^2-4\alpha^2\gamma^2} \qquad \ldots(8).$$

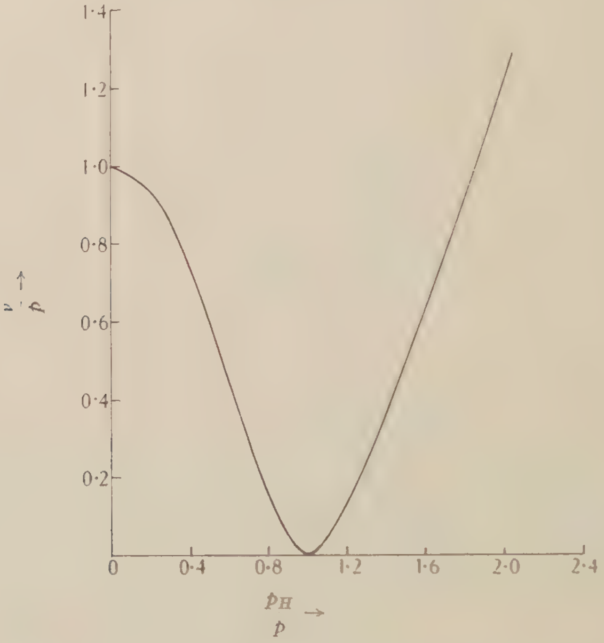

Figure 1.

Substituting from equations (2), (3) and (4) for α, β, γ, we have

$$\sigma=\frac{Ne^2}{m}\frac{\nu(p^2+\nu^2+p_H^2)}{(p^2+\nu^2+p_H^2)^2-4p^2p_H^2} \qquad \ldots\ldots(9).$$

From a practical point of view we are interested in the value of ν for which th expression becomes a maximum, the values of p and p_H being given. Unfortunate it is not possible to obtain this value of ν in a tractable algebraic expression, and w have therefore been compelled to use graphical methods. In figure 1 are given tl results of this analysis where the value of ν/p for maximum conductivity is plotte as a function of p_H/p.

This curve shows the very marked effect of p_H in determining the value of which produces maximum conductivity. The figure calls for two further comment It will be seen that when $p_H=0$, so that there is no applied magnetic field, the valu

$\nu=p$ for maximum conductivity as was shown above. It should also be noted at according to equation (9), from which the figure was derived, the value of ν which maximum conductivity is attained when $p_H/p=1$, is equal to zero. Since, wever, a value of ν equal to zero cannot be attained in a tube of finite size, it lows that we must expect divergences between the theory and the experimental sults when the applied magnetic field has the so-called critical value, i.e. when $=mcp/e$.

§ 3. THE EXPERIMENTAL ARRANGEMENTS

The method for investigating the variation of conductivity with pressure was e same as that used by Appleton and Chapman, the ionized medium being a w-pressure gas discharge. The discharge tube selected was 5 cm. in diameter d 50 cm. in length, cold aluminium disc electrodes 20 cm. apart being used. The plied voltage was supplied from an Evershed-Vignolles motor-generator with a turated diode in the supply circuit. The tube current and thus the magnitude of e ionization was conveniently controlled by varying the filament current of the ode.

The variation of conductivity was studied by the use of external plate electrodes tached to a Lecher-wire system in which was induced a resonant radio-frequency ectromotive force. This arrangement is illustrated in figure 2, where *T* is a section f the discharge tube, *LL* are the Lecher wires and *EE* the external plates. These lates which consisted of copper-foil sheets measured 6 × 4 cm. and were set cm. apart. A pair of Helmholtz coils was mounted coaxially with the tube so that uniform magnetic field could be imposed on that part of the discharge situated etween the external plates.

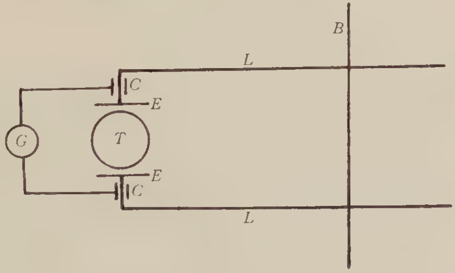

Figure 2.

The tuning-bridge *B* on the Lecher-wire system consisted of a rectangular opper plate measuring 30 × 20 cm. which could, by means of inlet tubes, be made o slide along the wires with its plane at right angles to their length. The entire pace near the free end of the Lecher system was shielded by means of a metal creen and carefully earthed. In this way the effect of the hand-capacity of the bserver was so reduced as to be negligible.

The amplitudes of the oscillations induced in the Lecher system from a neighbouring radio-frequency oscillator was, in the case of the longer wave-lengths used (i.e. from 80 to 260 cm.), measured in arbitrary units by means of a Duddell therm galvanometer coupled to the system by means of two small condensers *CC* co sisting of short brass tubes insulated from the Lecher wires by glass sleeves. F the range of wave-lengths studied triode oscillation generators differing in ty were used. Some were of the simple regenerative type while others produ internal electronic oscillations of the Barkhausen-Kurz type.

It was however considered desirable to test the method with extremely hi radio frequencies, corresponding to wave-lengths of from 15 to 40 cm., and that purpose a G.E.C. split-anode magnetron oscillator* was used and found work excellently. The Duddell thermogalvanometer was, however, not fou sufficiently sensitive for use with ultra-short waves. For these extremely hi frequencies we therefore used a thermocouple made by the Cambridge Scient Instrument Company and working with a sensitive mirror galvanometer.

In the part of the discharge opposite to the external plates there was fitte probe electrode for the measurement of electron-concentration by the meth described by Langmuir and Mott-Smith(6). By means of this we were able verify that when the pressure of the air in the tube was varied through the requi range, the value of N, the number of electrons per cm^3, did not vary so long as tube current was maintained by the diode at a constant value.

§ 4. EXPERIMENTAL PROCEDURE

The conductivity is shown by the damping which the presence of the discha produces in the oscillations induced in the Lecher-wire system. To study relation between conductivity and pressure, the resonance deflection of the therm galvanometer was first obtained by means of the tuning-plate bridge. The discha was then started for the minimum pressure, in the range to be investigated, a the resonance deflection was again noted. The pressure was then allowed increase in steps by means of a controlled leak, and resonance galvanome deflections were noted at each pressure. In all cases it was found that at particular pressure the amplitude of the oscillations was a minimum. This indica that the conductivity in the discharge was a maximum. Such measurements w made both without an imposed magnetic field and with an imposed field, magnitude of which was varied.

§ 5. EXPERIMENTAL RESULTS

In order to test the apparatus, measurements were made of the pressure wh produced maximum conductivity in the absence of an imposed magnetic fi The range of frequencies used was a little larger than that used by Appleton Chapman.

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* We are extremely indebted to the Director of the General Electric Company's Rese Laboratories for his kind assistance in this matter and to Mr E. C. S. Megaw of the same laborato for helpful advice on many points.

In figure 3 are shown some of the readings obtained. Here the resonance nplitude of the Lecher-wire system is shown as a function of the pressure in the scharge tube for three different wave-lengths. In each case it will be seen that e critical pressure is unaltered by a variation of N, brought about by a variation

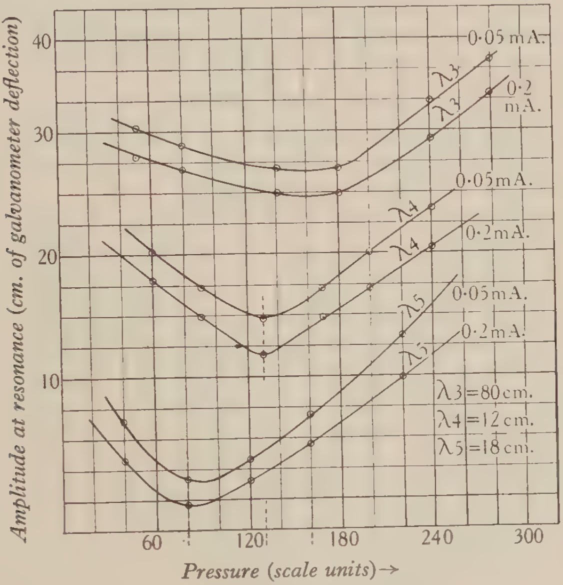

Figure 3.

f tube current, but that the critical pressure for maximum conductivity varies vith the wave-length.

In the subjoined table are tabulated the results obtained for the different wave-engths and the values of ν calculated from the relation $\nu = p$; see above, and articularly equation (7).

Table

Wave-lengths (cm.)	Frequency (c./sec. × 10^9)	Pressure X for maximum conductivity (mm. of mercury)	ν (sec.$^{-1}$ × 10^9)	$\frac{\nu}{X}$ (c.g.s.u. × 10^{10})
16	1·88	0·40	11·8	2·95
20	1·50	0·32	9·43	2·95
28	1·07	0·231	6·73	2·91
34	0·882	0·190	5·55	2·92
80	0·375	0·080	2·36	2·95
124	0·242	0·052	1·52	2·92
180	0·166	0·038	1·05	2·76
244	0·123	0·027	0·77	2·85
260	0·115	0·025	0·72	2·88

The constancy of the figures in the fifth column shows, as was expected, that the value of ν is proportional to the pressure.

In figure 4 are shown some of the readings obtained for various values of t imposed magnetic field, the frequency of the applied electromotive forces bei kept constant and corresponding to a wave-length of 80 cm. The necessity keeping the discharge conditions constant during the run rendered it impossil to make measurements at a large number of values of the imposed field. The valu chosen were $\frac{1}{2}H_0$, H_0, $\frac{3}{2}H_0$ and $2H_0$, where H_0 was equal to the critical field giv by the equation

$$p = p_H = H_0 e/mc.$$

H_0 for the case of 80-cm. waves was thus 133 gauss and corresponded to a curre of 4·43 amperes through the Helmholtz-coil system.*

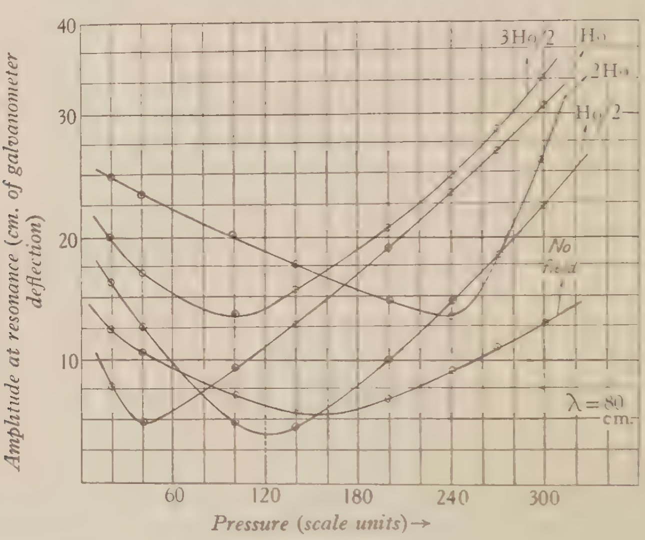

Figure 4.

It will be seen that the imposition of the field had a marked effect on t pressure at which the maximum conductivity was obtained. This is further illu trated in figure 5 where the critical pressure is exhibited as a function of t imposed field.

Now since ν is proportional to pressure we can compare the experimen curve in figure 5 with the theoretical curve in figure 1. Apart from certain poi which are discussed later, it will be seen that there is a striking similarity betwe the two curves. We can, therefore, conclude that both theory and experime indicate that the critical pressure at which the conductivity of an ionized mediu reaches its maximum value is very markedly altered by the imposition of a magne field in a direction at right angles to the applied alternating electric forces.

It will be noticed that the experimental results do not agree with the theo for the case in which the magnetic field attains the critical value. This disagreeme might be partially explained by the fact, previously mentioned, that under o xperimental conditions a zero value of ν is never attained. But we do not believe hat this is the only possible reason for the disagreement. It is also probable that, wing to the charges on the walls of the glass tube and the influence of positive-n sheaths, somewhat complex polarization forces are introduced. These may be xpected to be especially important when the quasi-resonance condition is reached and the electron amplitudes are large. We have attempted a quantitative treatment, taking into account the polarization effects of free charges appearing at the boundaries of a dielectric slab of limited extent, and found that in such a case the maximum conductivity for critical applied magnetic field would be expected at small but not zero values of ν; but as we are not sure that such theoretical premises accurately correspond with the experimental conditions we have not pursued the matter further.

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* Exactly comparable results, not given in the paper, were obtained for a wave-length of 260 c

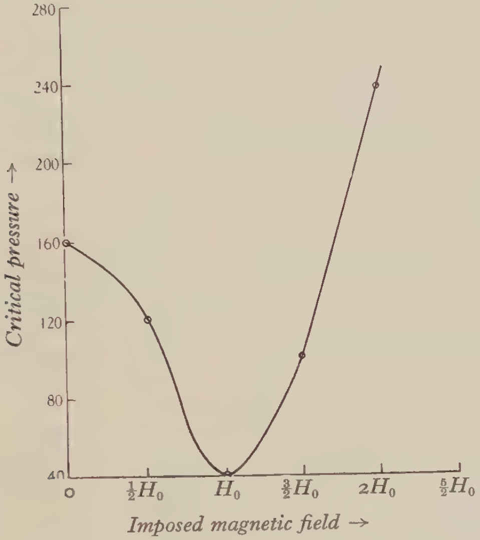

Figure 5.

§ 6. DISCUSSION OF RESULTS

The results summarized above have an important bearing on the current theories of the propagation of wireless waves in the ionosphere. The reflection of such waves by the upper atmospheric electricity is usually accompanied by a certain amount of absorption, and this absorption is attributed to the effects of the collision of electrons with the neutral air molecules. It has more than once been pointed out that the amount of absorption depends very largely on the amount of ionization at a certain critical level in the atmosphere, the height of which depends on the frequency of the waves in question. Thus, for example, if there is no imposed magnetic field the spatial attenuation coefficient κ is given by

$$\kappa=\frac{2\pi Ne^2}{c\mu m}\left(\frac{\nu}{\nu^2+p^2}\right) \qquad \ldots\ldots(10),$$

μ where μ is the refractive index of the medium for waves of angular frequency
Now the quantity in brackets on the right-hand side reaches a maximum valu
when $\nu=p$, so that if N/μ is not varying markedly with height there is a maximu
of absorption at the particular level in the ionosphere where this condition
attained(7).

It may, however, be shown that the existence of an imposed magnetic fiel
alters the height of this level of maximum attenuation just as it alters the pressu
at which maximum conductivity is attained, the two effects being merely differe
aspects of the same physical phenomenon. We take the two cases of propagati
along and at right angles to the field in order.

Case (a): propagation along the imposed magnetic field. Here we have corre
sponding to equation (10)

$$\kappa=\frac{2\pi Ne^2}{c\mu m}\left(\frac{\nu}{\nu^2+(p\pm p_H)^2}\right) \qquad \ldots\ldots(11),$$

where the upper and the lower signs of p_H correspond to the ordinary and extr
ordinary wave-components respectively. Under the conditions discussed abov
there is a maximum of absorption when

$$\nu=p\pm p_H \qquad \ldots\ldots(12).$$

Now as the value of p_H is comparable with the value of p commonly used in wirele
experiments we see that the height of maximum absorption is profoundly influence
by the earth's magnetic field.

Case (b): propagation at right angles to the imposed magnetic field. Here we mu
distinguish between the case in which the electric vector is parallel and that i
which it is at right angles to the imposed magnetic field. In the former case, of th
ordinary wave, there is no influence of the magnetic field on the wave-propagatio
and the equation (10) applies. In this case, therefore, the maximum absorptio
occurs when $\nu=p$. In the case of the extraordinary wave, where the electr
vector is at right angles to the magnetic field, we have, to the degree of approx
mation we have previously used in this paper,

$$\kappa=\frac{2\pi Ne^2}{c\mu m}\,\frac{\nu(\nu^2+p^2+p_H{}^2)}{(\nu^2+p^2+p_H{}^2)^2-4p^2p_H{}^2} \qquad \ldots\ldots(13).$$

Here the relation between the value of ν for maximum absorption and p_H
similar to that which has been made the subject of experiment in this paper, bein
shown theoretically in figure 1 and experimentally in figure 5. From these diagra
it may be deduced that for a large range of values of p_H/p the value of ν/p for maximum
absorption differs markedly from the value of unity which it assumes under th
condition of zero magnetic field.

A further word is necessary to emphasize the difference between the conditio
which obtain in the transmission of electric waves and those we have realized i
the laboratory. For the propagation of wireless extraordinary waves at right angl
to the imposed magnetic field the medium behaves as if the conductivity σ wer
given by

$$\sigma=\frac{p}{4\pi}\,\frac{\beta(1+2\alpha+\alpha^2+\beta^2+\gamma^2)}{(\alpha+\alpha^2-\beta^2-\gamma^2)^2+\beta^2(1+2\alpha)^2} \qquad \ldots\ldots(1a),$$

which reduces to equation (1) when the ionization is small and $|\alpha| \gg 1$. The difference between equations (1 *a*) and (1) is that the former takes account of the electric forces which arise in the direction of propagation and which are brought about by the longitudinal nature of the electronic motion. With a medium of size small compared with a wave-length, such as we have used in our laboratory experiments, we have neglected this effect since in any case it must be small in comparison with the local polarization forces arising from the fact that the medium s limited in extent. But as Lorentz has shown it should be taken into account in electric wave-propagation in an infinite medium and we believe that Pedersen, in his book *The Propagation of Radio Waves*, is unjustified in adopting equation (1) and specifically rejecting equation (1 *a*) as the appropriate expression for the effective conductivity in radio wave-propagation.

An allied topic of interest is the case of direct-current conductivities along and at right angles to the imposed magnetic field. The relevant formulae in this case were given by Townsend many years ago. They are easily derived from our formulae by putting p, the angular frequency, equal to zero. Thus for the conductivity σ_L along the field we have, from equation (7), σ_L

$$\sigma_L = \frac{Ne^2}{m\nu} \qquad \text{......(14).}$$

Correspondingly for the conductivity σ_T transverse to the magnetic field we have from equation (9), σ_T

$$\sigma_T = \frac{Ne^2}{m} \frac{\nu}{\nu^2 + p_H{}^2} \qquad \text{......(15).}$$

Equation (15) is of particular interest, for it indicates that, in higher magnetic atitudes, horizontal direct-current conductivity of the ionosphere is markedly nfluenced by the earth's magnetic field. The transverse conductivity per electron or per ion) reaches a maximum value when $\nu = p_H$. Now since p_H is equal to $\cdot 4 \times 10^6$ c./sec. for electrons and $1{\cdot}61 \times 10^2$ for, say, nitrogen molecules, it is clear hat there will be a level of maximum transverse conductivity per electron and another, higher, level of maximum transverse conductivity per ion. Now at these two levels of maximum conductivity we have

$$\sigma_T = \frac{Ne^2}{2mp_H} \qquad \text{......(16),}$$

or

$$\frac{\sigma_T}{N} = \frac{ec}{2H_0} \qquad \text{......(17),}$$

a result of great interest, for it indicates that the conductivity per electron or per ion is the same at the two levels of maximum conductivity. We thus see that the direct-current transverse conductivity due to ions can be just as important at a certain level as that due to electrons at a lower level, although in the case of the high-frequency conductivity the effects of the ions may be negligible in comparison with those of the electrons at all levels of the ionosphere.

§ 7. ACKNOWLEDGMENT

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Radio Research Board of the Department of Scientific and Industrial Research.

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MODULATION-FREQUENCY-CHANGE TECHNIQUE FOR IONOSPHERIC MEASUREMENTS

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ABSTRACT. The paper is a critical examination of Appleton's suggestion that the carrier-frequency-change technique should be modified by varying the frequency of modulation instead of the carrier frequency. Definite conclusions, which are set out in § 5 of the paper, have been reached.

§ 1. INTRODUCTION

FOUR experimental methods of estimating the effective heights of the ionized regions are known, namely (*a*) the frequency-change device of Appleton and Barnett[(2)]; (*b*) the group-retardation scheme of Breit and Tuve[(3)]; (*c*) the angle-of-incidence method of Appleton and Barnett[(4)]; (*d*) the moving-transmitter arrangement of Mirick and Hentschel[(5)]. It has been shown by Appleton[(1)] that the first two measure the equivalent path-difference between the ground and sky rays, while the others enable an estimate to be made of the angle of incidence of the sky ray at the ionosphere.

Appleton* has suggested, as especially suitable for ultra-short waves, a modification of the frequency-change method in which the amplitude of the transmitted wave is modulated at a variable audio-frequency but, so far as we are aware, no details have yet been given of the technique of ionospheric measurements made by this new method. In addition, it is not immediately obvious that the carrier-frequency-change and the modulation-frequency-change experiments should measure the same quantity.

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* See reference (6). The preliminary experiments were made in 1929 with positive results at the Peterborough Radio Research Station by Prof. Appleton and one of us (A. L. G.) by means of transmissions from the B.B.C. stations 2LO and 5SW at frequencies of 0·83 and 12 Mc./sec. An independent investigation was made in Australia by Dr W. G. Baker at about the same time, but the experiments were discontinued when it was learned that the matter was already receiving attention in England.

Further attempts to develop a satisfactory technique were initiated by Prof. Appleton and one of us (G. B.) in 1931, but the experiments were again delayed in view of the rapid development of the echo work[(7)].

The present technique is therefore closely related to previous work of both the British and the Australian Radio Research Boards, and in particular the new suppressed-carrier arrangement is the basis of much of our recent work on control of fading by special systems of modulation at the transmitting station.

The object of the present paper, therefore, is firstly to supply this deficiency and to bring the modulation-frequency-change device into line with the four basi experimental methods. Secondly it is desired to put forward a further modificatio of the modulation-frequency-change arrangement in which the carrier wave i suppressed at the transmitter, only the modulation sidebands being emitted
(b) *In what follows we refer to the suppressed-carrier modification as system* (*b*) *and t*
(a) *Appleton's modulation-frequency-change device as system* (*a*). It will be shown tha the former arrangement has some marked advantages over the latter, both in th actual technique of measurement and in the interpretation of results. Recen investigations⁽⁸⁾ involving the measurement of equivalent path-differences hav demonstrated the superiority of system (*b*) in this type of experiment.

§ 2. THEORY OF MODULATION-FREQUENCY-CHANGE EXPERIMENT, SYSTEM (*a*)

Before proceeding to the investigation of system (*a*), it will be convenient, fo the purpose of comparison, to summarize very briefly the salient features of th
E_0, E_s carrier-frequency-change experiment. If E_0, E_s are the vertical components o
r the electric intensities of the ground and sky rays of radio-angular frequency r
θ and θ is the phase-difference between them, then the signal at the receiver is

$$E_0 \sin rt + 2E_s \sin (rt+\theta) \qquad \text{......(1)},$$

where

$$\theta = -\frac{rD}{c} \qquad \text{......(2)},$$

D, c D is the optical path-difference between the ground and sky rays, and c is th velocity of electromagnetic radiation in free space.

When a frequency-change of amount Δr is made at the transmitter, the receive
n signal passes successively through a number Δn of interference fringes, determine by

$$\Delta n = -\frac{\Delta r}{2\pi}\cdot\frac{d\theta}{dr} = \frac{\Delta r}{2\pi c}\cdot\left(D + r\cdot\frac{dD}{dr}\right)$$
$$= \frac{D'}{c}\cdot\frac{\Delta r}{2\pi} \qquad \text{......(3)},$$

D' where D' is defined as the equivalent path-difference and is the quantity measure in the experiment.

Further, if the receiver includes a square-law detector and the maximum an minimum deflections of the galvanometer in its anode circuit are denoted re
I, m, A spectively by M^2 and m^2, then we define a quantity A which determines the ampli tude of the fringes to be such that

$$A = \frac{M^2 - m^2}{M^2 + m^2} = \frac{2a}{1+a^2} \qquad \text{......(4)},$$

a where a is written for $2E_s/E_0$. From equations (3) and (4) it is therefore possible t calculate the equivalent path of the sky ray and the relative intensities of the groun and sky rays.

In the modulation-frequency-change system (a), three waves are simultaneously emitted and the signal at the receiver due to the ground rays is

$$E_0\left[\sin rt+\frac{Q}{2}\{\sin(r+q)\,t+\sin(r-q)\,t\}\right]$$

which

$$=E_0\,(1+Q\cos qt)\sin rt \qquad \text{......(5)},$$

where q is the angular frequency of modulation of depth Q. The sky-ray signals at the receiver are q, Q

$$2E_s\left[\sin(rt+\theta)+\frac{Q}{2}\left[\sin\{(r+q)\,t+\theta'\}+\sin\{(r-q)\,t+\theta''\}\right]\right] \qquad \text{......(6)},$$

where

$$\theta'=\theta+q\frac{d\theta}{dr}=\theta+\chi,$$

θ', χ

$$\theta''=\theta-q\frac{d\theta}{dr}=\theta-\chi \qquad \text{......(7)},$$

θ''

and, from equation (2),

$$\chi=q\frac{d\theta}{dr}=-q\frac{D'}{c} \qquad \text{......(8)}.$$

Hence, the sky-ray signals may also be written as

$$2E_s\,\{1+Q\cos(qt+\chi)\}\sin(rt+\theta) \qquad \text{......(9)},$$

which is to be compared with equations (1) and (5) above.

After square-law detection of the combined ground and sky-ray signals, the d.-c. components in the galvanometer are proportional to

$$(E_0^2+4E_s^2)\,(2+Q^2)+4E_0E_s\,(2+Q^2\cos\chi)\cos\theta \qquad \text{......(10)},$$

in which it should be noted that θ is invariant with respect to q.

As the frequency of modulation is changed, $\cos\chi$ varies between the limits ± 1 and the number of fringes Δn produced by a change Δq of modulation angular frequency is given by

$$\Delta n=-\frac{\Delta q}{2\pi}\cdot\frac{d\chi}{dq}=\frac{D'}{c}\cdot\frac{\Delta q}{2\pi} \qquad \text{......(11)}.$$

On comparing this result with equation (3), it will be found that both the carrier-frequency-change and the modulation-frequency-change experiments measure the equivalent path-difference D'. However, in the latter case the amplitude of the observed fringes is A_1 where A_1

$$A_1=\frac{2a\,Q^2\cos\theta}{(1+a^2)\,(2+Q^2)+4a\cos\theta} \qquad \text{......(12)},$$

from which it follows that A_1 depends not only on the sky-ray intensity, but also on the depth of modulation and on the instantaneous phases of the ground and sky rays. Even under the most favourable experimental conditions it is found that the fringe-amplitude is very much less than that to be observed with the carrier-frequency-change technique and, when the phase-difference between the ground and sky rays is such that $\cos\theta=0$, the fringes are completely eliminated.

In practice it is desirable to make use of an artifice* which will facilitate th delineation of the fringes. The required effect has been achieved by passing th audio-frequency signal components from the anode circuit of the detector to second square-law rectifier to which the galvanometer is connected.

From equations (5) and (9) it may be shown that the audio-frequency curren components in the first detector are proportional to

$$E_0^2\,(4Q\cos qt+Q^2\cos 2qt)+4E_s^2\,\{4Q\cos(qt+\chi)+Q^2\cos 2\,(qt+\chi)\}$$
$$+4E_0E_s\cos\theta\left\{4Q\cos\frac{\chi}{2}\cos\left(qt+\frac{\chi}{2}\right)+Q^2\cos(2qt+\chi)\right\}\quad\text{......(13)},$$

and, after rectification of this signal in the second detector, the d.-c. component passing through the galvanometer are proportional to

$$Q^2\,[1+a^4+2a^2\cos\chi+2a\cos\theta\,(1+\cos\chi)\,(a\cos\theta+1+a^2)]$$
$$+\frac{Q^4}{16}[1+a^4+2a^2\cos 2\chi+4a\cos\theta\,\{a\cos\theta+(1+a^2)\cos\chi\}]\quad\text{......(14)}.$$

It will be observed that the harmonic distortion in the first detector, of frequenc $2q$, has given rise to a term containing $\cos 2\chi$ which will produce spurious secondar fringes on the records. However, the relative amplitudes of the subsidiary to th primary fringes are proportional to $Q^2/16$ and, in practice, it is found that the effec is negligible if the depth of modulation at the transmitter is limited to values of th order of 30 per cent.

With this condition fulfilled, the number of fringes is determined solely b variations of $\cos\chi$ and is in fact given by equation (11), as for the first detecto On the other hand, the fringe-amplitude is now A_2 where

A_2

$$A_2=\frac{4a\,(a+\cos\theta)\,(1+a\cos\theta)}{(1-a^2)^2+(1+a^2+2a\cos\theta)^2}\quad\text{......(15)},$$

and, except for the two special cases in which the intensity and phase of the sky ra are such that $\cos\theta=-a$ or $-1/a$, the fringes are easily distinguished. In fact when $\cos\theta=+1$, equation (15) reduces to $A_2=2a/(1+a^2)$, showing that in thi case the fringe-amplitude is identical with that which would have been observe with carrier-frequency-change technique. It should also be noticed that A_2 i independent of the depth of modulation at the transmitter, whereas the fringe amplitude in the first detector is directly proportional to Q^2.

Of the two special cases cited above in which the fringe-amplitude falls to zer only the first is of practical importance. The second condition clearly can only b satisfied when the sky-ray signal is greater than that of the ground ray, and this i usually avoided in frequency-change experiments.

An alternative equation which has been found somewhat more convenient than equation (15) for calculation, is derived in the following way. We extract th square roots M and m of the maximum and minimum galvanometer-deflections and define a quantity F as a measure* of the signal fluctuations, thus

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* In the preliminary experiments carried out by Prof. Appleton at the Peterborough Radi Research Station the same principle was made use of.

$$F=\frac{M-m}{M+m}=\frac{a\,(a+\cos\theta)}{1+a\cos\theta} \qquad \text{......(16).}$$

This equation should be used with care if it is suspected that the sky-ray signal is greater than that of the ground ray. Signal-fluctuations are greatest when $\cos\theta=+1$ and when $a=1$; there are no signal-variations when $a=0$ or $-\cos\theta$, in accordance with equation (15) above.

§ 3. THEORY OF MODULATION-FREQUENCY-CHANGE EXPERIMENT, SYSTEM (*b*)

Measurements carried out with system (*a*) have disclosed a number of difficulties in the technique of the receiving-apparatus, of which the most serious is the extreme susceptibility to interference by atmospherics and by adjacent continuous-wave transmissions. Both types of interference were found to be considerably enhanced by the use of the second detector in the receiver, and the suggestion to suppress the carrier at the transmitter was made as a result of a search for a method which would obviate the necessity of employing successive rectifiers. It was realized that the amplitude of the fringes after one stage of detection would then be sufficiently great for recording purposes and, indeed, subsequent work has shown that when system (*b*) is used no advantage is to be expected from the addition of the second detector. Since it is not necessary to resupply the carrier at the receiver it is possible to use the same apparatus for modulation-frequency-change experiments as is required in carrier-frequency-change technique.

In system (*b*) only the modulation sidebands are emitted and the signal at the receiver due to the ground rays is proportional to

$$\frac{E_0}{2}\{\sin(r+q)\,t+\sin(r-q)\,t\}=E_0\cos qt\sin rt \qquad \text{......(17),}$$

while the corresponding sky-ray signal is

$$E_s\,[\sin\{(r+q)\,t+(\theta+\chi)\}+\sin\{(r-q)\,t+(\theta-\chi)\}]$$

which

$$=2E_s\cos(qt+\chi)\sin(rt+\theta) \qquad \text{......(18).}$$

This is to be compared with equations (1), (5) and (9) above.

In the first detector the d.-c. components which actuate the galvanometer are proportional to

$$E_0^2+4E_s^2+4E_0E_s\cos\theta\cos\chi \qquad \text{......(19).}$$

Thus the number of fringes produced by a change in the modulation frequency is determined by variations of $\cos\chi$, and the equivalent path-difference between the ground and sky rays may be computed, as in the case of system (*a*), from equation (11), thus

$$\Delta n=\frac{\Delta q}{2\pi}\cdot\frac{D'}{c}.$$

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* In the carrier-frequency-change experiments it is often convenient to use F in preference to A. For comparison with equation (4) above, $F=a$ under the same conditions.

It is important to notice, however, that the amplitude of the fringes is now give very simply by the equation

$$A = \frac{2a \cos \theta}{1 + a^2} \qquad \text{......(20).}$$

Hence when the instantaneous phase between the ground and sky rays is such tha $\cos \theta = 1$, the amplitude of the fringes is as great as would have been obtained wit carrier-frequency-change technique. At other times there is a definite reductio in fringe-amplitude and, in particular, no fringes are observable when $\cos \theta = 0$.

§ 4. EXPERIMENTAL RESULTS

A number of comparisons have been made of ionospheric heights by the carrie frequency-change and the modulation-frequency-change techniques at a mea frequency of 1·45 Mc./sec. The receiver was located at Liverpool, New Sou Wales, distant 25 kilometres from the emitter at Sydney University. In all case substantial agreement has been found between the two methods, subject to exper mental errors in the measurement of the amounts of the frequency-change in t two techniques.

For example, on October 1, 1934, when the carrier-frequency-change wa 6300 c./sec. and the modulation frequency was varied between the limits 900 an 6300 c./sec., pairs of records were obtained at short intervals starting from 10 p.m E.S.T., and the equivalent heights computed were as shown in the table.

Time, p.m. (E.S.T.)	10.00	10.10	10.20	10.25	10.30	10.35	10.40
Heights by carrier-frequency-change	107	107	258	256	260	255	262
Heights by modulation-frequency-change	109	109	262	262	251	253	260

It will be observed that a jump occurred from the *E* to the *F* region at 10.15 p.m and that the two series of measurements were in good agreement for both region

On another occasion, September 28, 1934, measurements were made durin the sunset period during which there was a steady increase of height of the region, followed by a jump to the *F* region at 6 p.m. Sunset on this day was 5·55 p.m. The results of the measurements by the two methods are shown i figure 1, from which an idea may be gained of the order of agreement to be expecte between the modulation-frequency-change and the carrier-frequency-chang techniques.

A careful check has also been made of the manner in which the amplitude the fringes in the modulation-frequency-change experiment depends both on th intensity and on the instantaneous phase relationships of the ground and sky rays Figures 2 and 3 show the results, for a typical set of experimental conditions, whic would be expected to follow from the theory previously given. In both cases th experimental conditions correspond to reflection of sky rays from the *E* regio at nearly vertical incidence, and it has been assumed that $D' = 225$ km. and $a = 0{\cdot}5$. Points in the curves have been computed for a change of modulation frequency from 500 to 2000 c./sec., but the extensions required for greater frequency-changes will be easily visualized.

Figure 2, for modulation-frequency-change system (*a*), has been computed by substituting a range of values of θ in expression (15). The critical phase angle between the ground and sky rays which determines the elimination of fringes is given by $\cos\theta = -a$, and is 120° for the experimental conditions chosen. When the ground- and sky-ray signals corresponding to the carrier are in phase, $\theta = 0°$ and

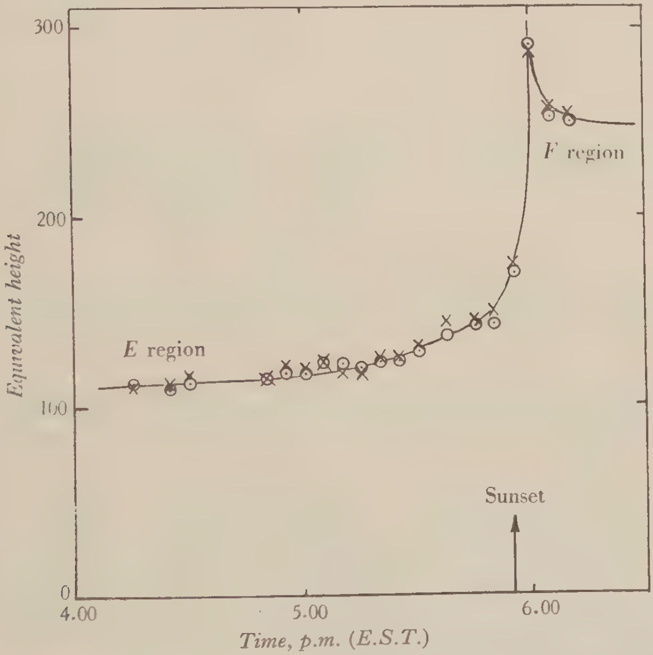


Figure 1. Comparison of ionospheric heights measured by modulation-frequency-change of 5370 c./sec. ⊙ ⊙, and carrier-frequency-change of 6000 c./sec. × ×.

the fringe-amplitude is a maximum and is as great as would have been observed in a carrier-frequency-change experiment. For increasing values of θ the fringe-amplitude steadily decreases through zero until finally there is a reversal in sense. However, in the special case when $a = 1$ there is no reversal, but both the fringe-amplitude and the mean signal attain zero simultaneously. It should be noted that the beginning of the fringe occurs in every case at the same point of a cycle, independently of the instantaneous value of θ. This result is in striking contrast with the carrier-frequency-change case in which the commencement of the artificial fading cycle is determined by the momentary state of the natural fading.

Figure 3, for modulation-frequency-change system (*b*), is calculated from equation (20). In this case the fringes always have zero amplitude when $\theta = 90°$, independently of the intensity of the sky ray. The fringe-amplitude is a maximum and equal to that observed in a carrier-frequency-change experiment both whe $\theta = 0°$ and 180°. Apart from the reversal in sense as θ passes through 90°, th commencement of each fringe is at the same point in the cycle, independently o the phase of ground and sky rays.

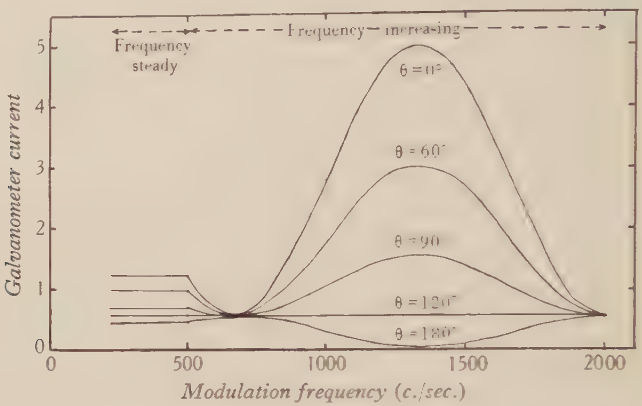


Figure 2. Theoretical variation of fringe-amplitude in the modulation-frequency-change metho with normal modulation, compared with the radio-frequency phase angle θ between the groun and sky rays.

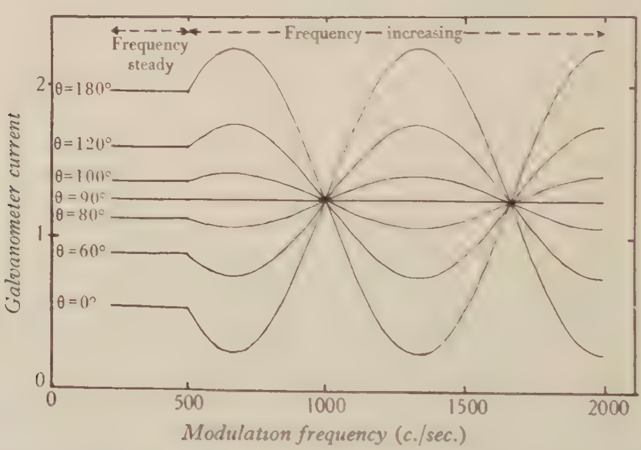
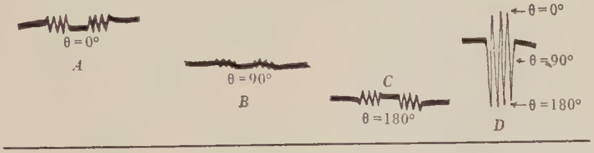
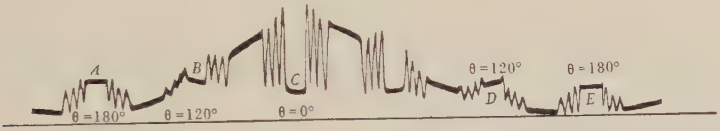


Figure 3. Theoretical variation of fringe-amplitude in the modulation-frequency-change metho with suppressed carrier.

Drawings from typical records chosen to illustrate these points are shown i figures 4, 5 and 6. In figure 4 there are three sets of fringes from the same recor of the anode current in the first detector, for system (*a*) and a depth of modulatio of about 30 per cent at the transmitter. It is seen that the fringe-amplitude wa always small and, at the point marked *B*, very nearly zero; at that time the phas angle between ground and sky rays was 90°. At *A* and *C*, when the state of th atural fading showed that the two rays were respectively in phase and in antiphase, ie fringe-amplitude was at a maximum. The interesting sense-reversal of the inges, which is predictable from equation (12), is apparent on comparison of and *C*. For this record the modulation-frequency-change was 5·5 kc./sec., so hat the 3·8 fringes indicated an equivalent path-difference of 207 km. A set of fringes btained under comparable conditions but with the carrier-frequency-change ystem are shown at *D*, and the comparison shows clearly the need for the second etector in the modulation-frequency-change experiments when system (*a*) is sed.



igure 4. Experimental comparison of interference fringes obtained with carrier-frequency-change and modulation-frequency-change with normal modulation. *A*, *B*, *C*, first detector in system (*a*); *D*, carrier-frequency-change system.



igure 5. Experimental verification of variation of fringe-amplitude with radio-frequency phase angle θ between ground and sky waves. Modulation-frequency-change with normal modulation, was employed and the fringes were recorded in the output of the second detector.

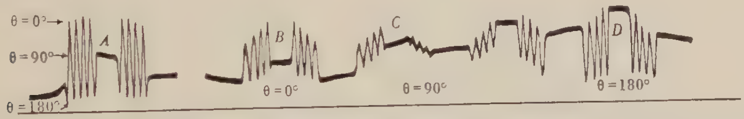


Figure 6. Comparison of carrier-frequency-change method. *A*, with modulation-frequency-change; *B*, *C*, *D*, suppressed-carrier method.

Figure 5 is for the second detector in tests made with modulation-frequency-change system (*a*). Six complete sets of fringes are shown, each comprising an increase and a decrease of modulation frequency of amount 5·5 kc./sec. At *C*, in the middle of the record, the ground and sky rays were in phase and the fringe-amplitude was a maximum. At *A* and *E*, the beginning and end of the record, the ground and sky rays were in antiphase and the fringes had undergone the sense-reversal predicted in figure 2 from equation (15). It is known* that the modulation frequency was steady and at its minimum value at the points marked *A*, *B*, *C*, *D*, *E*, and it may be seen from the record that the sense of the fringes was such that the received signal at *C* at first increased as the modulation frequency began to increase,

* The apparatus which produces the frequency-changes is so arranged that there is a short pause of about 1 sec. duration following the decrease of frequency and a longer pause of about 2 sec. after the frequency-increase.

whereas at A and E frequency-increases were accompanied by decreases of signa During the time that this record was being taken, natural fading was occurring a rate somewhat too fast to enable the reduction of fringe-amplitude to zero to shown throughout a complete set of fringes. However, the transition, correspondi to cos $\theta = -a$, is clearly observed at B and it seems likely that the phase-differen between ground and sky rays was about 120° at this time, in agreement wi equation (15).

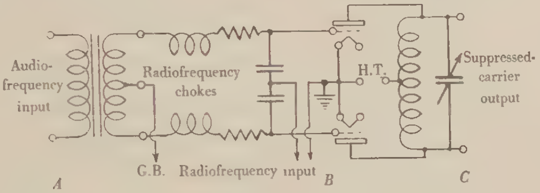
Figure 6 is for the first detector in modulation-frequency-change system (At A are shown, for comparison, a set of fringes taken with the carrier-frequenc change arrangement 1 min. previously. The reduction of fringe-amplitude to ze and the reversal in sense of the fringes occurred at C, at which time the phas difference between ground and sky rays was presumably 90°, according to equati (20). At B, when $\theta = 0°$, the short pause was followed by an increase of frequen and by an increase of signal, whereas at D, when $\theta = 180°$, the fringe immediat following the short pause began with a decrease of signal-intensity.

Many other records have been obtained with both systems (a) and (b), a agreement has always been found with the theory given here. It is therefore cle that modulation-frequency-change suffers at least one considerable disadvanta when compared with carrier-frequency-change, namely that the amplitude of t fringes is, on the average, much less with the former technique. Another exper mentally verified peculiarity of the modulation-frequency-change system, that t phase at the beginning of a set of fringes is substantially independent of t instantaneous state of the natural fading, has an important bearing on measureme of polarization and will be discussed later.

§ 5. DISCUSSION AND CONCLUSIONS

Although this paper is mainly concerned with the technique of ionospher measurements by means of modulation-frequency-change systems it does n appear to be necessary to describe the apparatus in great detail. We are satisfi that system (b) is the more satisfactory of the two modulation-frequency-chan arrangements, so that if this system is used it is unnecessary to make any alteratio to receiving-apparatus which has already been set up for carrier-frequency-chan work. As to the transmitter, it is impossible to use the simple arrangement co prising a self-excited oscillator which has hitherto proved adequate for most carri frequency-change experiments. However, the substitution of a low-power mast oscillator and amplifying stages has been found to be an improvement in transmitt technique for carrier-frequency-change work, so that when once this replaceme has been made it is an easy matter to arrange for carrier-suppression. The syste which we have found useful is shown in figure 7. The output from a beat-frequen audio-frequency oscillator is applied in push-pull to the grids of two valves throu a transformer at A, whereas the carrier is supplied at B to the two valves in phas radio-frequency chokes and resistances serve to isolate radio- and audio-frequen rcuits. When the valves are carefully matched the output at C consists only of the odulation sidebands, since the carrier is balanced out and the output tuned rcuit presents negligible impedance to currents of audio frequency.

It has already been pointed out that the phases of the beginnings and ends of e fringes in a modulation-frequency-change experiment are, with the exception the phase-reversal, independent of the momentary phase relations of the ground d sky rays. On the other hand, a knowledge of these phases in carrier-frequency-ange work has made possible the measurement of the polarization(9, 10) of the y ray, of lateral deviation(11, 12), and of changes in optical path(13). The following alysis shows that the modulation-frequency-change system is not suitable for ese experiments and, in fact, it is easy to see that its use is very largely limited to easurements of equivalent path.



igure 7. Carrier-suppression unit. The circuit is adapted from carrier telephony practice for use at high radio frequencies.

Consider the signals received in a rotatable loop, figure 8. It will only be ecessary to work out the case for system (b), since similar results hold for system (a). f the plane of the loop makes an angle ϕ with the plane of propagation, then the ϕ

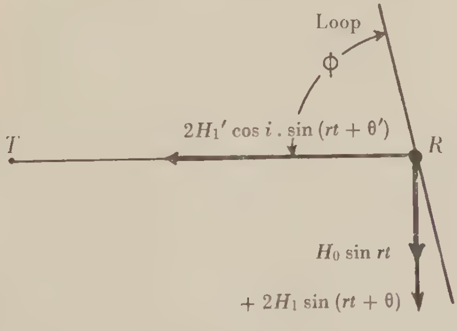

Loop
ϕ
$2H_1' \cos i \,.\, \sin(rt+\theta')$
T
R
$H_0 \sin rt$
$+\,2H_1 \sin(rt+\theta)$

Figure 8. Ground plan of magnetic vectors of ground and sky waves.

signal due to the reception of a ground ray H_0 and an elliptically polarized sky ray H_0 H_1, H_1', arriving at an angle of incidence i, is H_1, H_1

$$\begin{aligned}&H_0 \cos\phi \cos qt \sin rt\\ &\quad +2H_1 \cos\phi \cos(qt+\chi)\sin(rt+\theta)\\ &\quad +2H_1' \cos i \sin\phi \cos(qt+\chi)\sin(rt+\theta') \qquad \ldots\ldots(21),\end{aligned}$$

if we assume that the polarization of sky rays of frequency $(r+q)$ and $(r-q)$ is not very different from that at frequency r. In that case the polarization of the down-

R_c coming ray is determined by the ratio R_c of components, which $=(H_1'\;H_1)\cos$
ξ and their phase-difference ξ which $=\theta'-\theta$.

The signal in the loop may be rewritten as

$$H_0 \cos\phi \cos qt \sin rt + 2H_s \cos(qt+\chi)\sin(rt+\theta_s) \qquad \ldots\ldots(22),$$

H_s where

$$H_s = H_1 \cos\phi\,\sqrt{(1+R_c^2\tan^2\phi + 2R_c\tan\phi\cos\xi)} \qquad \ldots\ldots(23),$$

θ_s and

$$\tan\theta_s = \frac{\sin\theta + R_c\tan\phi\sin\theta'}{\cos\theta + R_c\tan\phi\cos\theta'}.$$

It should be remembered that both H_s and θ_s are invariant with respect to chang in the modulation frequency. The similarity in form of equations (22), (17) and (1 shows that the currents passing through the galvanometer after square-law de tection of the signals are proportional to

$$H_0^2\cos^2\phi + 4H_s^2 + 4H_0H_s\cos\phi\cos\theta_s\cos\chi \qquad (24),$$

from which it follows immediately that, although the amplitude of the fring depends on H_s and θ_s, i.e. on the polarization of the downcoming ray, the fring phases are determined by χ, and are therefore independent of the polarization an of the orientation of the loop aerial. Hence, any experimental method of deter mining polarization, such as that due to Appleton and Ratcliffe, which depends fo its success on recognizing variations in fringe-phases in differently oriented re ceiving aerials, is inapplicable to the modulation-frequency-change technique.

Similar considerations rule out the use of modulation-frequency-change system in the measurement of lateral deviation and changes in optical paths of the sky ray It is perhaps to be emphasized that we have tacitly assumed throughout that th receiving apparatus is sufficiently broadly tuned to be capable of receiving both the sidebands sent out from the transmitter. However, it has been suggested th there may be advantages in some cases in observing the two signals in separat sharply tuned receivers. With this technique it has been found possible, for exampl to emit two waves with frequencies differing by 40 kc. sec. but with known relativ phases, and thus to compare changes in the optical paths of the two sky rays.

The conclusions to be drawn from this comparison of the carrier-frequency change and the modulation-frequency-change techniques are as follows:

(i) The number of fringes produced by a change of modulation frequency i proportional to the equivalent path of the sky ray, as in the case of a carrier-frequency change.

(ii) The amplitude of the fringes in modulation-frequency-change is very muc less than in carrier-frequency-change, unless an artifice such as the two square-la detectors in Appleton's system (*a*), or the device of suppressing the carrier at th transmitter in system (*b*), is used.

(iii) After either of these modifications have been made, the fringe-amplitud is occasionally as great as would have been obtained with the carrier-frequency change technique. There are certain relations, however, between the carrier phase and amplitudes of the ground and sky rays for which the modulation-frequency change fringes have zero amplitude, and on the average the carrier-frequency-change fringes always have the greater amplitude.

(iv) In the modulation-frequency-change experiment, the phase at the beginning of a set of fringes is independent of the momentary state of the natural fading. It is also independent of the state of polarization of the sky ray and of the orientation of the receiving loop aerial. Hence, modulation-frequency-change technique is inapplicable to experiments where fringe-phases are important; for example measurements of polarization, lateral deviation, or variations of optical path.

(v) A comparison of modulation-frequency-change systems (*a*) and (*b*) suggests that the latter is the more satisfactory technique for ionospheric measurements. There is little to choose between the two methods in regard to the modifications which would need to be made to a carrier-frequency-change transmitter, but there are marked advantages in system (*b*) at the receiver, both in reception technique and in the interpretation of records.

§ 6. ACKNOWLEDGMENTS

The work described here formed a part of the programme of the Radio Research Board of the Australian Commonwealth Council for Scientific and Industrial Research, to which the authors are indebted for permission to publish their results. They are also indebted to Mr A. H. Mutton and Dr O. O. Pulley for assistance at the transmitter, to Dr D. F. Martyn for criticism of the paper, and to Prof. J. P. V. Madsen of the P. N. Russell School of Engineering of the University of Sydney for much help and advice during the course of the investigation.

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A RECEIVER DISCRIMINATING BETWEEN RIGHT AND LEFT-HAND CIRCULARLY POLARIZED WIRELESS WAVES

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ABSTRACT. A brief review of the history of magneto-ionic effects in the ionosphe and the apparatus developed for their investigation is given. Because of the inflexibili of present methods of determining the polarization of received waves, a new scheme h been developed which permits observations to be made over a range of frequencies with out readjustment. The principle employed is that of an effectively rotating loop, th rotation being simulated by the modulation in phase quadrature of two loops at right angles. The theory of operation, the errors and the method of adjustment are investigate Adjustment can be made with the aid of a test wave of unknown polarization.

Some results obtained with the apparatus are discussed, and it is concluded that th two magneto-ionic components are almost circularly polarized, and that the relativ intensities are unpredictable unless the complete (P', f) curve appropriate to the time observation is available. A Fourier-series treatment of the propagation of a pulse energy of radio frequency in the ionosphere is given in an appendix.

§ 1. HISTORICAL

As soon as an attempt was made to use wireless waves for the purpose determining position by measurement of the direction of arrival of suc waves from a known station, it became evident that this direction, as foun from the position of a loop aerial when the received signal was a maximum or minimum, differed from the expected or true direction, and furthermore, that th difference varied with time. To account for this, it was suggested by Bellini(1) an by T. L. Eckersley(2) that the error is connected with the presence of an abnormall polarized component in the received wave, i.e. a component in which the vector electric force does not lie in the vertical plane through sender and receiver. T explain the presence of such abnormally polarized waves, the magneto-ionic theory was put forward by Appleton(3), who showed that if the effective agents in th upper atmospheric reflection were of electronic mass, the effect of the earth' magnetic field would be to cause double refraction and rotation of the plane o polarization. Further work has amply demonstrated the correctness of Appleton' theory.

Smith-Rose and Barfield(4) made a systematical study of the effect by comparin the relative signal-amplitudes from various combinations of loop and vertica aerials. Later, Appleton and Ratcliffe⁽⁵⁾ obtained direct confirmation of the theory, which was further strengthened by the results obtained by Green⁽⁶⁾ working under similar conditions but in the southern hemisphere. The method used was for studying the variation of signal-intensity as measured on three loops, one in the plane of propagation and the other two making angles of ± 45° with this direction, when the frequency of the transmitted waves was varied. About the same time, Hollingworth⁽⁷⁾ made a similar investigation on the longer wave-lengths with apparatus which was slightly simpler but required less acceptable assumptions.

With the adoption of the pulse method of investigation, Appleton and Builder⁽⁸⁾ showed the composite nature of the echoes from the upper region, and interpreted the doublets as being due to magneto-ionic splitting. Apparatus was then developed at the Radio Research Board at Slough⁽⁹⁾ for demonstrating the polarization of the components of the doublets. The method consists of applying the output, suitably amplified, from two loops at right-angles to pairs of plates in a cathode-ray oscillograph. The resulting Lissajou's figure is an indication of the state of polarization of the received wave, and a simple test determines the sense.

Almost simultaneously, J. A. Ratcliffe and E. L. C. White⁽¹⁰⁾ developed their receiver for circularly polarized waves, in which two loops at right angles are coupled through a mutual inductance. When the receiver is suitably adjusted, a circularly polarized wave having one sense of rotation produces an e.m.f. at the grid of the amplifying valves, while one of opposite sense produces no effect. Reversal of the sense of the mutual coupling permits the reception of the wave polarized in the opposite sense while suppressing the original one. T. L. Eckersley⁽¹¹⁾ originated a somewhat similar scheme, but detuned the two loops in opposite directions in order to effect the cancellation of one sense of polarization.

The results in all cases were in agreement with Appleton's magneto-ionic theory as far as the actual polarization of the doublets was concerned, though some differences of opinion as to the relative intensities of the two components still exist.⁽¹²⁾

German workers⁽¹³⁾ have investigated the problem from the point of view of the Fourier components of the pulse, and so deal with continuous waves of various frequencies rather than with a discrete train of waves of a single frequency. Assuming the magneto-ionic theory to account for a path-difference between the two circularly polarized components of a wave of given frequency, they observe that the plane of polarization of the resultant wave at the receiver varies with frequency. The summation of these component plane-polarized waves, with due allowance for the rotation of the plane of polarization with frequency, results in a complex pulse, usually a doublet. It is shown in appendix C that this doublet is the resultant of two pulses, identical in shape with the emitted pulse, and both circularly polarized, but that when the two pulses overlap the polarization varies throughout the overlapping region, and the shape of the resultant pulse bears only an indirect relation to the shape of the emitted wave since the two components have to be added vectorially. The result is quite independent of frequency dispersion and, incidentally, this factor will not adequately account for the spreading or broadening of echoes of one polarization found experimentally. Both the theoretical and experimental results of Handel and Plendl merely confirm the accepted magneto-ionic theory.

The magneto-ionic theory, then, is well established and is of considerable importance, not only in the transmission of wireless waves but also in several branches of geophysics. But a slight uncertainty with regard to certain points in itself justifies another approach to the problem, and the inflexibility, with respect to frequency-change, of all present schemes makes such an undertaking very desirable.

§ 2. DISCUSSION OF AVAILABLE SCHEMES

All schemes at present in use for the determination of the state of polarization of wireless waves require considerable care in adjustment and this adjustment is necessary for each frequency. In their present state it would not be possible to make observations on more than one frequency in, say, 5 min., and if a range of frequencies sufficient to yield a curve of the variation of height with frequency had to be covered, then conditions at the end of the run would be quite different from those at the beginning. The two-receiver method of the Radio Research Board is very convenient for visual observation, but is unsuitable for recording purposes. It is expensive, since two separate receivers are necessary; furthermore, the constancy of adjustment is likely to be poor, since there are many circuits in which variations can occur.

For recording purposes, the polarization ellipse must be reduced to its elements. These elements might be the magnitude of the major and minor axes, their angle with respect to a reference line, and the sense in which the ellipse is described. Alternatively, the magnitude of the two circular components having opposite senses of rotation and their phase relation might be used. Since it is to be expected that the polarization of the reflected waves will be nearly circular, the latter scheme is preferable; the phase relation is usually omitted for simplicity.

The methods of Ratcliffe and White(10) and of Eckersley(11) can be used for recording the two circular components but, apart from their inflexibility, it is necessary to assume that the testing wave is linearly polarized; as yet, no technique has been developed for adjustment when this wave is, of necessity, elliptically polarized.

All methods are very critical in adjustment, for they depend on the phase of the currents in the loops, which is much more sensitive to incorrect tuning than is the amplitude-adjustment. When a change to another frequency is made, an adjustment other than retuning is necessary—in one case in the amount of coupling, and in the other in the amount of detuning. Finally, in both of these schemes, when it is desired to change from the suppression of one sense of rotation to the suppression of the other, it is necessary to perform a switching operation in a circuit carrying currents of radio frequency, and it is difficult to ensure that circuit conditions shall not be altered in the process.

§ 3. A SUGGESTED SCHEME

If a loop rotates in a clockwise or right-hand direction when viewed from above, then it is obvious that a circularly polarized wave of right-hand sense of rotation will produce in the loop an e.m.f. of frequency less than that of the incident wave, whereas one of left-hand sense will produce an e.m.f. of greater frequency. When applied to a superheterodyne receiver these e.m.fs. produce, in the intermediate-frequency amplifier, corresponding e.m.fs. the frequencies of which, whatever the signal frequency, differ by twice the frequency of rotation of the loop. They can then be separated by fixed band-pass filters, and the means of discrimination between the two senses of rotation is independent of the signal frequency. With continuous waves, this rotation of the loop might be purely physical, but with pulses in which the Fourier component frequencies may cover a range of 10 kc./sec. or more, rotational frequency of the loop must be greater than this figure, or it will not be possible to separate the two components. Such a speed of rotation can only be produced by an artifice. Remembering that a rotating vector can be considered as the sum of two simple harmonic vectors in time and phase quadrature, it is seen that one method is to cause the output from two loops at right-angles to vary sinusoidally but in phase quadrature and to add the two outputs. Grid modulation with variable-mu valves can be made suitably linear and of the correct form, i.e. $\cos \omega t \cos pt$ and not $\cos \omega t\,(1+\cos pt)$, by applying the signal and modulating e.m.fs. to the grids in push-pull and connecting the anodes in parallel. After this stage of modulation (or of superheterodyne detection as it may be called if the matter is regarded from another point of view) the resultant frequencies are passed to any ordinary pulse-receiver and the two component frequencies are separated as if they were different signals. A convenient check is available in that the direction of rotation of the equivalent loop can be reversed, so that either sense of rotation can be received at either frequency. For automatic recording, the upper frequency is chosen arbitrarily, and either right- or left-hand polarization is received by reversing the sense of rotation of the loop. No radio-frequency currents flow through this reversing switch, for the operation is performed in the modulating circuit, and the adjustment of the high-frequency circuits is unaffected.

Such a system has been developed in the Halley Stewart laboratory and is described and investigated in detail below.

§ 4. DESCRIPTION OF THE APPARATUS

The schematic diagram of the apparatus is shown in figure 1. On the left-hand side is the modulating oscillator, the frequency of which determines the frequency of rotation of the effective loop. This frequency is about 33 kc./sec., well above the highest modulation frequencies of the pulse, and the particular value is chosen so that convenient commercial values of capacity and resistance (0·01 μF. and 500 Ω.) may be used in the bridge modulating circuit. Balance to earth is essential throughout the apparatus, so the oscillator is in push-pull disposition as shown. Shunt feed to the anodes permits the bridge circuit to be directly connected to the output of the oscillator. The bias provided by the resistance-capacity circuit R_{12}, C_{12}, is adjusted to give the purest wave-form to the output, since harmonics give rise to unwanted frequencies in the output from the modulator. The inductances L_1 are balanced to earth. The main frequency-adjustment is effected by means of the condenser C_9. The output is taken from across two relatively large condensers C_{10}, of 0·02 μF. capacity to ensure good balance to earth. The resistances R_{10} provide earth connexions for the grids of the valves. This method of coupling reduces the harmonic content of the output to a negligible value.

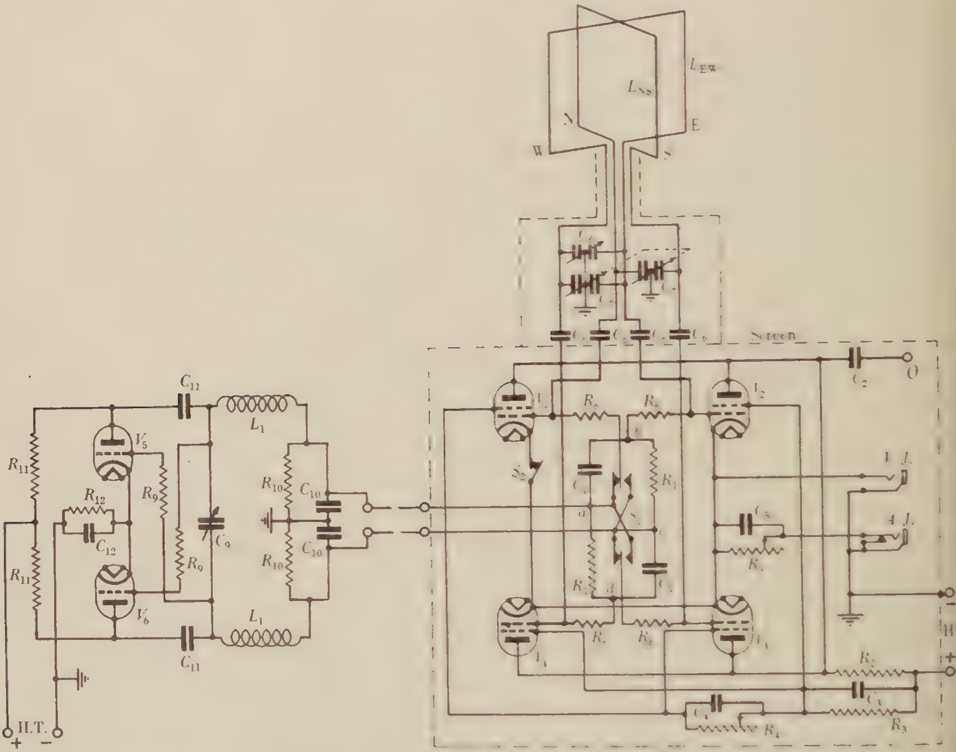


Figure 1. Circular polarization receiver.

The output is taken to the two corners a, c, of the resistance-capacity bridge circuit R_1, C_1, R_1, C_1. When the frequency of the oscillator is correctly adjusted, the voltage across the two pairs of diagonals V_{ac} and V_{bd} are equal and 90° out of phase, and the voltages of the opposite points V_a and V_c, or V_b and V_d, of either diagonal are 180° out of phase with reference to earth-potential.

The grids of the valves V_1, V_2, V_3, V_4 are connected each to one corner of the bridge circuit, and the bias on the valves is adjusted by means of the resistance R_5, which is shunted by smoothing-condenser C_5, to obtain a modulating bias of the correct form. The anodes are connected together and the output from the set goes through the condenser C_2 to the first circuit of a superheterodyne receiver normally used for pulse-reception.

Mounted on top of the screened box containing the modulating stage are the mutually perpendicular loops L_{NS} and L_{EW}; four leads are taken from these to the top compartment of the box and connected each to one section of a four-gang condenser having a capacity of 0·0005 μF. per section. Since the moving plates and spindle of this condenser are earthed, the loop circuit is suitably balanced, and trimming-condensers correct this balance. An auxiliary trimming-condenser C_8 ensures that the two loops shall be exactly in tune with each other. Each of the four leads from the loops is also connected to one of the grids of the modulating valves, through a stopping-condenser C_6 of capacity 0·0001 μF. This condenser presents a high impedance to currents of modulation frequency, which would otherwise be short-circuited through the loops. Similarly, resistances R_6 in the leads to the grids from the bridge circuit present a high impedance to radio-frequency currents.

Other necessary components in the set are shown. The jacks *VJ* and *AJ* are for checking the steady bias and the total anode current respectively. The resistance R_2 is the anode feed impedance; C_2 is the output coupling condenser of capacity 0·0001 μF. When the system is operating correctly, currents of modulation frequency should be entirely absent from the anode circuit, so that filtering is not essential. For the same reason, the dropping-resistance for the screen voltages is smoothed by a capacity of only 0·05 μF. A further dropping-resistance R_4 is included in the lead to one pair of valves, so that the maximum amplification of each pair can be adjusted to equality; the current through this resistance is no longer constant and a smoothing-condenser of 1 μF. is required. A switch S_1 reverses the connexions from the bridge circuit to the valves V_1 and V_3 so that, in effect, the direction of rotation of the equivalent loop is reversed. Another switch S_2 opens the cathode circuit of one of the valves and thus unbalances the set sufficiently to produce currents of signal frequency in the anode circuit when desired. The whole apparatus is arranged to be as symmetrical as possible about a vertical axis in order to prevent coupling between the two loops and their amplifiers, and to reduce unavoidable unbalance to a minimum.

The loops are wound with bronze wire, for mechanical reasons. The extra resistance introduced makes the electrical adjustments less critical. The inductances of the two loops are adjusted to equality by means of ebonite bridges which strain together the wires of the loop.

§ 5. THEORY OF OPERATION

Curve *a* of figure 2 shows the variation of the mutual conductance with grid-bias for a typical variable-mu valve (VMS_4). In both pairs of valves in the modulating stage the grids are in push-pull and the anodes in parallel, so to find the curve for the resultant mutual conductance curve *b* must be added to curve *a*. By suitably adjusting the steady bias on the two valves the sum of these two curves, curve *c*, can be made practically linear over a range of ± 5·5 V. about the mean bias of − 5·5 V. The load impedance of the stage is small so that the mutual conductance

μ is a measure of the amplification, so that $\mu = kE_g$, where μ is the amplification
k, E_g factor of the stage, k is a constant, and E_g is the voltage on the grid, measured from
E_1 the point of steady bias. If, in addition, a small voltage E_1 is impressed on the
grid, it will produce a voltage E_1kE_g across the anode circuit. If E_g is replaced by
t, E, ω the modulating voltage $V\cos pt$, and E_1 is replaced by the signal voltage $E\cos\omega t$,
then the voltage in the anode circuit becomes $kVE\cos pt\cos\omega t$.

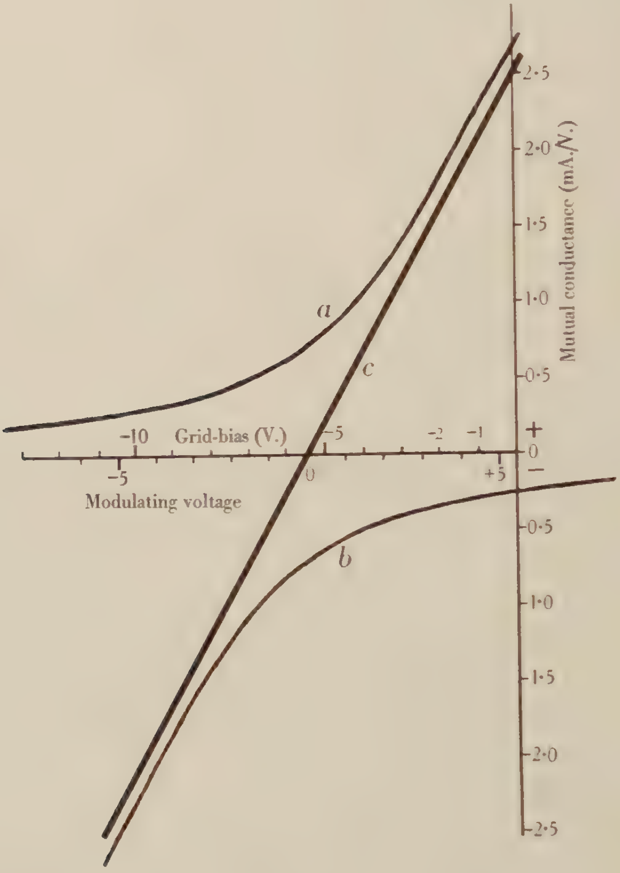


Figure 2.

Now in the other pair of valves the modulating voltage is $E\sin pt$, and if the incident signal wave is circularly polarized, E_1 becomes $\pm E\sin\omega t$, according to the sense of rotation. The resultant voltage across the anode circuit due to all four valves then is

$$kVE(\cos\omega t\cos pt \pm \sin\omega t\sin pt)$$

which

$$= kVE\cos(\omega \pm p)t,$$

i.e. the frequency is either increased or decreased by an amount equal to the modulation frequency, according to the sense of rotation of the incident circularly polarized wave. The signal frequency is not present, and also, if the sense of rotation of the equivalent loop is reversed, that circular component which produced currents at the upper frequency now produces currents at the lower frequency, and *vice-versa*.

In general, whatever the state of polarization of the incident wave or its angle of incidence, the projection of the magnetic vector on to the horizontal plane will be the only portion that can affect the loops, and this projected vector can in turn be resolved into two rotating vectors of opposite sense. One of these rotating vectors will produce in the anode circuit currents of frequency equal to the sum, and the other will produce currents of frequency equal to the difference, of the signal and modulating frequencies. The currents or voltages will be truly proportional to the magnitudes of the two resolved vectors, and a measurement of these currents is a measurement of the state of polarization of the horizontal component of the incident wave.

Since no currents of signal frequency appear in the anode circuit, and the main amplifier is tuned to one or other of the two resultant frequencies, pick-up in the leads from the modulator stage to the main amplifier introduces no errors. It is therefore feasible to locate the loops and modulator at a considerable distance from the amplifier and recording apparatus, if this is desired.

§6. ERRORS AND ADJUSTMENTS

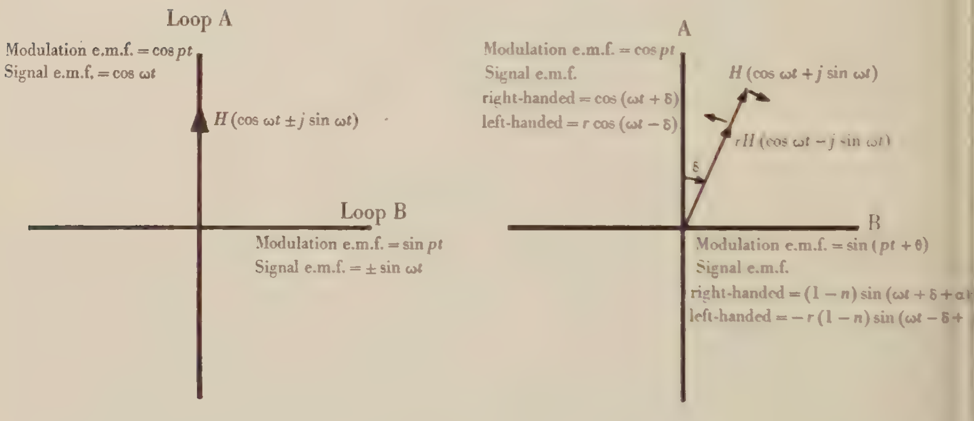
Only the right-handed and left-handed rotating vectors of the horizontal component of the magnetic field of the incident wave will be considered, with the usual convention that the sense of rotation is referred to an observer looking along the direction of propagation of the wave before reflection at the ground—i.e. looking downwards. The reference modulation voltage is $\cos pt$ and the reference signal voltage is $\cos \omega t$ so that the right-handed and left-handed rotating vectors are $(\cos \omega t + j \sin \omega t)$ and $(\cos \omega t - j \sin \omega t)$ respectively. At zero time, the signal reference vector makes an angle δ with that loop in which the modulation is $\cos pt$. δ
Conditions, then, are as shown in figure 3*a*.

Owing to errors in the bridge circuit, the two modulating voltages may not be exactly 90° out of phase, so $\sin pt$ is replaced by $\sin(pt+\theta)$. If correctly tuned to resonance, the voltages induced in loop A by the right and left-handed vectors are $\cos(\omega t+\delta)$ and $\cos(\omega t-\delta)$ respectively, while those induced in loop B are $\sin(\omega t+\delta)$ and $-\sin(\omega t-\delta)$ respectively. Even if the two loops are not correctly in tune, but are detuned by equal amounts in the same direction, i.e. if they have the same phase angle, it is still possible to use these two expressions for the induced voltages, if zero time is suitably chosen. But if the phase angles differ by, say, an amount α, then $\sin(\omega t+\delta)$ and $-\sin(\omega t-\delta)$ must be written as $\sin(\omega t+\delta+\alpha)$ and α
$-\sin(\omega t-\delta+\alpha)$ respectively. Another source of error is that either the modulation voltages on the two loops, the voltages induced in the loops for a given signal strength, or the amplifications produced by the two pairs of valves may not be equal. Since the voltage in the anode circuit is the product of all three, it is not possible to separate the various effects. It is sufficient to say that the output from

loop B is $(1-n)$ times that from loop A. Conditions, then, are as shown in figure 3*b*. The resultant effect due to a component of right-hand sense is

$$\cos(\omega t+\delta)\cos pt+(1-n)\sin(\omega t+\delta+\alpha)\sin(pt+\theta)$$

$$\begin{aligned}\text{which} &= \tfrac{1}{2}\left[\cos\{(\omega+p)t+\delta\}+\cos\{(\omega-p)t+\delta\}\right.\\ &\qquad \left.+(1-n)\left(\cos\{(\omega-p)t+\delta+\alpha-\theta\}-\cos\{(\omega+p)t+\delta+\alpha+\theta\}\right)\right]\\ &=\left(1-\frac{n}{2}\right)\cos\left\{(\omega-p)t+\delta+\frac{\alpha-\theta}{2}\right\}\cos\frac{\alpha-\theta}{2}\\ &\qquad+\frac{n}{2}\sin\left\{(\omega-p)t+\delta+\frac{\alpha-\theta}{2}\right\}\sin\frac{\alpha-\theta}{2}\\ &\qquad+\left(1-\frac{n}{2}\right)\sin\left\{(\omega+p)t+\delta+\frac{\alpha+\theta}{2}\right\}\sin\frac{\alpha+\theta}{2}\\ &\qquad+\frac{n}{2}\cos\left\{(\omega+p)t+\delta+\frac{\alpha+\theta}{2}\right\}\cos\frac{\alpha+\theta}{2} \qquad\text{......(1).}\end{aligned}$$


Loop A
Modulation e.m.f. = cos pt
Signal e.m.f. = cos ωt
H (cos ωt ± j sin ωt)
Loop B
Modulation e.m.f. = sin pt
Signal e.m.f. = ± sin ωt
A
Modulation e.m.f. = cos pt
Signal e.m.f.
right-handed = cos (ωt + δ)
left-handed = r cos (ωt − δ)
H (cos ωt + j sin ωt)
rH (cos ωt − j sin ωt)
δ
B
Modulation e.m.f. = sin (pt + θ)
Signal e.m.f.
right-handed = (1 − n) sin (ωt + δ + α
left-handed = − r (1 − n) sin (ωt − δ +

(*a*) (*b*)

Figure 3.

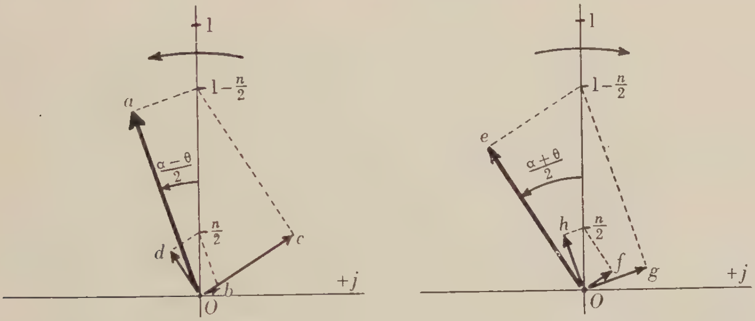
The resultant effect due to a component of left-hand sense is

$$\cos(\omega t-\delta)\cos pt-\sin(\omega t-\delta+\alpha)\sin(pt+\theta)$$

$$\begin{aligned}&=\left(1-\frac{n}{2}\right)\cos\left\{(\omega+p)t-\delta+\frac{\alpha+\theta}{2}\right\}\cos\frac{\alpha+\theta}{2}\\ &\qquad+\frac{n}{2}\sin\left\{(\omega+p)t-\delta+\frac{\alpha+\theta}{2}\right\}\sin\frac{\alpha+\theta}{2}\\ &\qquad+\left(1-\frac{n}{2}\right)\sin\left\{(\omega-p)t-\delta+\frac{\alpha-\theta}{2}\right\}\sin\frac{\alpha-\theta}{2}\\ &\qquad+\frac{n}{2}\cos\left\{(\omega-p)t-\delta+\frac{\alpha-\theta}{2}\right\}\cos\frac{\alpha-\theta}{2} \qquad\text{......(2).}\end{aligned}$$

Both right and left-hand components produce effects at the upper and lower frequencies and it remains to be seen just how the various effects can be made obvious. Any apparatus which is sensitive only to a uniformly rotating vector of constant amplitude can have no sense of orientation, in other words the effect produced in the apparatus must not vary as it is rotated about the axis of rotation of the vector—in this case the vertical axis. It is to be expected that errors will affect this axial symmetry. By varying δ in the above expressions the errors should be made apparent.

The vectors representing the terms in the above expressions are shown in figure 4 and their direction of rotation as δ is increased positively is counterclockwise or clockwise according as the terms contain $+\delta$ or $-\delta$, as is indicated in the figure. The experimental procedure is to observe the effects at a single frequency, so the vectors are regrouped as in figure 5 where, in both diagrams, they are of the same frequency and can be added graphically. They rotate in opposite directions as δ is varied, so that this addition has to be performed for each value of δ. Terms containing $\frac{n}{2}\sin\frac{1}{2}(\alpha\pm\theta)$ are neglected as being of the second order of small quantities. To make conditions quite general, an arbitrary state of polarization is assumed, such that the left-hand component is r times the right-hand one.



$$Oa=\left(1-\frac{n}{2}\right)\cos\left\{(\omega-p)\,t+\delta+\frac{\alpha-\theta}{2}\right\}\cos\frac{\alpha-\theta}{2};\qquad Oe=\left(1-\frac{n}{2}\right)\cos\left\{(\omega+p)\,t-\delta+\frac{\alpha+\theta}{2}\right\}\cos\frac{\alpha+\theta}{2};$$

$$Ob=\frac{n}{2}\sin\left\{(\omega-p)\,t+\delta+\frac{\alpha-\theta}{2}\right\}\sin\frac{\alpha-\theta}{2};\qquad Of=\frac{n}{2}\sin\left\{(\omega+p)\,t-\delta+\frac{\alpha+\theta}{2}\right\}\sin\frac{\alpha+\theta}{2};$$

$$Oc=\left(1-\frac{n}{2}\right)\sin\left\{(\omega+p)\,t+\delta+\frac{\alpha+\theta}{2}\right\}\sin\frac{\alpha+\theta}{2};\qquad Og=\left(1-\frac{n}{2}\right)\sin\left\{(\omega-p)\,t-\delta+\frac{\alpha-\theta}{2}\right\}\sin\frac{\alpha-\theta}{2};$$

$$Od=\frac{n}{2}\cos\left\{(\omega+p)\,t+\delta+\frac{\alpha+\theta}{2}\right\}\cos\frac{\alpha+\theta}{2};\qquad Oh=\frac{n}{2}\cos\left\{(\omega-p)\,t-\delta+\frac{\alpha-\theta}{2}\right\}\cos\frac{\alpha-\theta}{2}.$$

Figure 4.

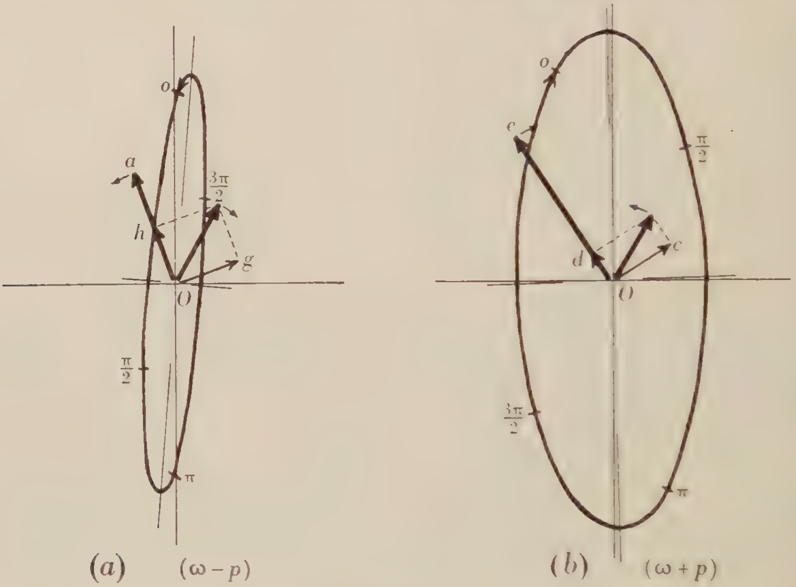
The equations corresponding to figure 5 can be written thus. For figure 5*a* the resultant effect at frequency $(\omega-p)$

$$\begin{aligned}=&\left(1-\frac{n}{2}\right)\cos\left\{(\omega-p)\,t+\delta+\frac{\alpha-\theta}{2}\right\}\cos\frac{\alpha-\theta}{2}\\ &+\left(1-\frac{n}{2}\right)r\sin\left\{(\omega-p)\,t-\delta+\frac{\alpha-\theta}{2}\right\}\sin\frac{\alpha-\theta}{2}\\ &+\frac{nr}{2}\cos\left\{(\omega-p)\,t-\delta+\frac{\alpha-\theta}{2}\right\}\cos\frac{\alpha-\theta}{2}\qquad\text{......(3).}\end{aligned}$$

For figure 5*b*, the resultant effect at frequency $(\omega+p)$

$$=r\left(1-\frac{n}{2}\right)\cos\left\{(\omega+p)\,t-\delta+\frac{\alpha+\theta}{2}\right\}\cos\frac{\alpha+\theta}{2}$$
$$+\left(1-\frac{n}{2}\right)\sin\left\{(\omega+p)\,t+\delta+\frac{\alpha+\theta}{2}\right\}\sin\frac{\alpha+\theta}{2}$$
$$+\frac{n}{2}\cos\left\{(\omega+p)\,t+\delta+\frac{\alpha+\theta}{2}\right\}\cos\frac{\alpha+\theta}{2} \qquad \ldots\ldots(4).$$

The ellipses of figure 5 show the variation of output as the loop assembly is rotated, and noting that δ is measured in opposite directions round the two curves (upon which values of δ are marked) we see that the major axes are almost at right-angles when α is large. As α is decreased, the eccentricity approaches zero and th major axes approach each other, one ellipse having passed through the circula form in the process. Variation of θ and n in the correct direction brings the eccentricity still closer to zero until finally both curves become circles and there is no variation of output as the loops are rotated.



(*a*) $(\omega-p)$ (*b*) $(\omega+p)$

Figure 5.

This, then, constitutes the technique of adjustment and it is evident that n special state of polarization of the testing wave is required. It is inconvenient t retune to find the effects at the two frequencies, but, as has been noted above, th same effect can be produced by altering the direction of rotation of the equivalent loop, i.e. by operating the switch S_1. The set can be adjusted by means of a loca oscillator, which does not have to be accurately tuned to the desired signal; no does it have to be situated at any great distance from the apparatus, several yards being quite sufficient.

The method of operation is as follows. The modulator and main amplifier are tuned to the signal frequency by unbalancing the former, by operating switch S_2, with the modulating oscillator switched off. On closure of S_2 and switching on of the oscillator, the signal can be found at two settings of the heterodyne oscillator, corresponding to the two frequencies $(\omega \pm p)$.* One of these frequencies is selected arbitrarily. The loop assembly is rotated and the positions at which the maxima and minima occur are noted, first with the switch S_1 in one position so that the relay is unoperated, and then in the other so that the relay is operated. If the minima are nearly at right angles to each other in the two cases, the trimming-condenser C_8 is adjusted in such a way as to bring the positions of minima together, and one ellipse should pass through the circular form in the process. Then, by varying condenser C_9 of the oscillator, the variation of output which occurs as the loops are rotated can be decreased, and by adjustment of the resistance R_4 the residual variation can be still further decreased. After a repetition of this series of adjustments, the variation for both settings should be negligible.

This adjustment is made at the high-frequency end of the range of the set and then repeated for the low frequency end; but in this case the loops are tuned to each other, α being thus reduced, by altering their inductance by means of the ebonite bridges, instead of by alterating the trimming-condenser. The residual variation that occurs as the loops are rotated can be reduced below ± 2 per cent, and it is then not symmetrical, owing to detuning of the loop circuits as the leads therefrom twist on rotation. When the adjustment is made at both ends of the frequency range, it is found that, for any setting of the tuning condenser, the error is less than ± 5 per cent; this figure can always be reduced to ± 2 per cent by slight readjustment at the required frequency. Thus it becomes feasible to use the set to determine the state of polarization of the received wave at a series of frequencies in rapid succession, for the whole set can be tuned in the same manner as an ordinary receiver, and the time taken for a complete $\{P', f\}$ curve is not more than twice that required for its delineation by the usual manual methods. If accuracy better than ± 5 per cent is required considerably more time would have to be taken.

There are a number of other errors which may occur, but they can be tested for and eliminated independently. Among these is the effect of harmonics in the modulation stage, with which can be grouped the distortion in the valves. Harmonics can only produce effects at frequencies $(\omega \pm 2p)$, $(\omega \pm 3p)$ ..., and such effects will be filtered out in the intermediate stages of the receiver. They introduce no errors into the measurements, but merely impair the efficiency of the apparatus and raise the general noise-level. Antenna effect may be present, but any voltages due to this cause will presumably be in the same phase on both grids of one pair of valves so that, to an approximation, the resultant modulation will be of the form $\cos \omega t\,(1 + \cos pt)$ and the terms in $(\omega \pm p)$ will have only half the amplitude of those due to a loop voltage of the same magnitude; thus there is convenient discrimination. By careful attention to symmetry in construction and by the use o push-pull modulation, antenna effect can be made negligible.

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* There are, of course, two other settings associated with the second channel of the superheterodyne receiver.

Mutual coupling between the two loops may exist, but it follows from th treatment of Ratcliffe and White(10) that its effect is to increase the output due t one sense of rotation and to decrease that due to the other by equal amounts, whil the output from either sense remains constant as the loops are rotated; the mai adjustments are not affected. Denoting the outputs due to the circular component in the absence of this error by R and L and the diminution due to coupling by we find that the resultant outputs are $(R+c)$ and $(L-c)$. The sum is $(R+L$ which is the output from a single loop in the position of maximum signal and i evidently independent of the error. The difference is $(R-L+2c)$ and the outpu from a single loop in the minimum position is $(R-L)$. By comparing thes quantities, the presence of mutual coupling between the loops can be detected, an the test is independent of the state of polarization of the testing wave. The chec should be made at the high-frequency end of the range since the mutual effec increases with frequency if, as is most common, the coupling is inductive.

R, L, c

Appendixes A and B deal respectively with the magnitude of the errors to expected from difference in phase angle in the two loops, and with variation frequency of the modulating oscillator. The latter effect is not serious but th former is extremely critical and, unfortunately, is present in all types of polarizatio analysers. The effect may be minimized by damping the loop circuits, but set noi soon limits this artifice. With dipole antennae in place of loops this limitati would not be so serious, on account of the greater signal voltage available, and might even be feasible to dispense entirely with tuned circuits in the modulat stage. The scheme could then be adapted for the automatic registration of $\{P',$ curves for one component only.

The principal disadvantage of the scheme presented is that it is impossible obtain any high degree of preselection in the single tuned circuit of the modulat stage, since the resultant frequencies are only separated by twice the modulati frequency, i.e. by about 60 kc. sec. As a result the signal-to-noise ratio is lessene In order to improve conditions, the modulation frequency might be raised and th would probably be justified, in spite of the slightly greater difficulty incurred the adjustment of the bridge circuit and in the elimination of the spurious effe due to switching. Care must be taken that neither fundamental nor harmon frequencies fall within the pass band of the intermediate-frequency filter.

§ 7. ADVANTAGES AND DISADVANTAGES OF THE SYSTEM

Advantages. (i) The system can be adjusted by means of a wave of unknow polarization. (ii) The frequency of the testing-wave need not be accurately equal that of the required signal. (iii) The means of discrimination between the two sens of rotation is independent of the signal frequency. (iv) The operation of changi the sense of rotation to be received is performed in a circuit carrying currents fixed and relatively low frequency. (v) The tuning-controls can be so ganged as

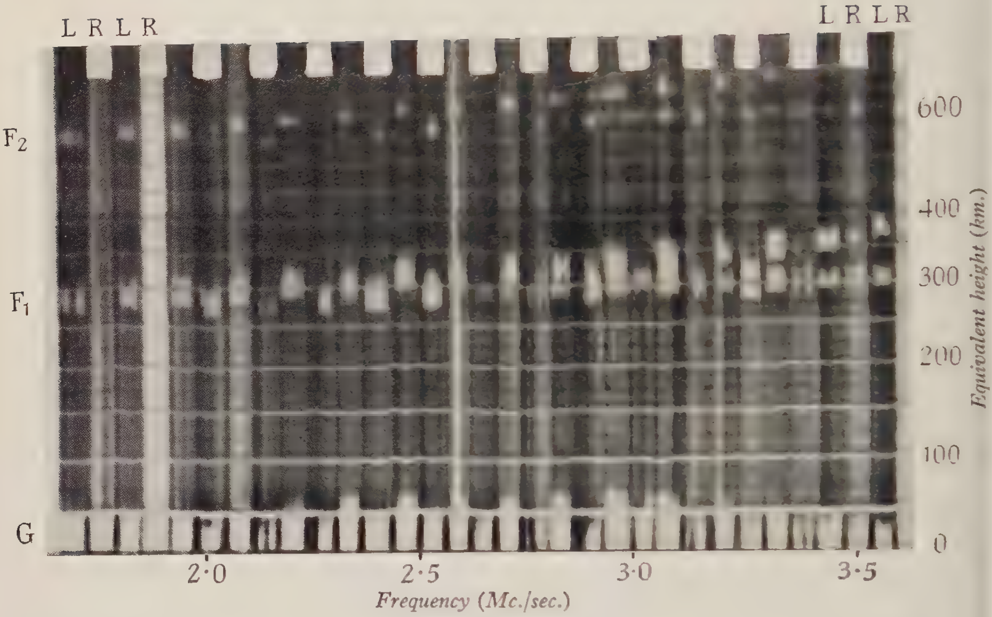
L R L R
L R L R
F₂
F₁
G
600
400
300
200
100
0
Equivalent height (km.)
2·0
2·5
3·0
3·5
Frequency (Mc./sec.)

Figure 6.

ermit the reception of a range of frequencies without readjustment. (vi) The ystem can be used aperiodically. (vii) Physical variation of the circuit constants s, in general, self-compensating.

Disadvantages. (i) The signal-to-noise ratio is lower than in other systems. ii) The system is somewhat complicated.

§ 8. RESULTS

Figure 6 shows a curve of equivalent height against frequency taken with the apparatus just described. The transmitted frequency has been varied in small steps and alternate frequencies were received with the loop sensitive to one sense of polarization only; the sense is indicated at each step by markings on the upper edge of the diagram. The base line only of the time base is recorded, so that only if the wave is truly circularly polarized does an echo appear for a single sense. Polarization is seldom accurately circular and in general there will be some effect for both senses, as in the case shown in figure 6, but a comparison of the relative amplitudes gives immediately the eccentricity and the sense of rotation.

Owing to facilities being limited, the set has been operated actually inside the laboratory and the distortion of the polarization due to conductors in the building is quite marked at certain frequencies. Thus it is imperative that the apparatus should be capable of being adjusted by means of a wave of unknown polarization. For the same reason we cannot rely on the measured values of eccentricity, but the distortion is never so serious as to lead us to confuse the senses of rotation except for almost linear polarizations.

The record shown was taken in the same time as the usual $\{P', f\}$ curves, i.e. 30 sec. at each frequency, and the set was handled exactly as if it were a normal receiver. No adjustment of the set, other than the necessary tuning was attempted, during the run. The frequency-range covered in the record is from 1·7 to 3·6 Mc./sec. and is roughly the critical region for the Appleton region at this particular time (midnight). It is seen that the two components are very nearly circularly polarized with opposite senses of rotation, at the lower frequencies. The eccentricity departs somewhat from zero as the frequency increases but distortion due to the building becomes apparent at about 3·5 Mc./sec.; even at the end of the record, however, there is a considerable difference between the amplitudes of one component in the two senses.

From the records taken, it seems quite impossible to state accurately the relative intensities of the two components, but the evidence confirms the theoretical result that the component having the lower critical frequency is approximately circularly polarized with a left-hand sense of rotation, and that having the greater critical frequency is also nearly circularly polarized, but with a right-hand sense of rotation.* The records have not been sufficiently numerous to lead to any definite conclusions as to the relative intensities, but with the evidence available from a very extensive series of $\{P', f\}$ curves, it appears that the intensities are quite unpredictable unless the complete $\{P', f\}$ curve for a particular time is available also. The fact that either component may have the greater time delay has long since been accepted. The more complex structure of the ionosphere, which has been revealed as the technique of $\{P', f\}$ measurements has improved, makes it possible that the absorption of either component may increase or decrease with frequency.

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* Only the range of frequencies above 1·7 Mc./sec. has been under investigation.

Take, for example, the curves of figure 7, which are of a type commonly encountered. The upper curves show the variation of equivalent height of the ordinary and the extraordinary rays for the range of frequencies from 2·4 to 6·0 Mc. sec Polarization measurements have been made showing that these rays are approximately circularly polarized with opposite senses of rotation, the ordinary being left-handed and the extraordinary right-handed. Below these curves are the corresponding curves for the estimated intensities of the two components. I is seen that the relative amplitudes depend very markedly on the particular fre quency under investigation, and either component may be the greater.

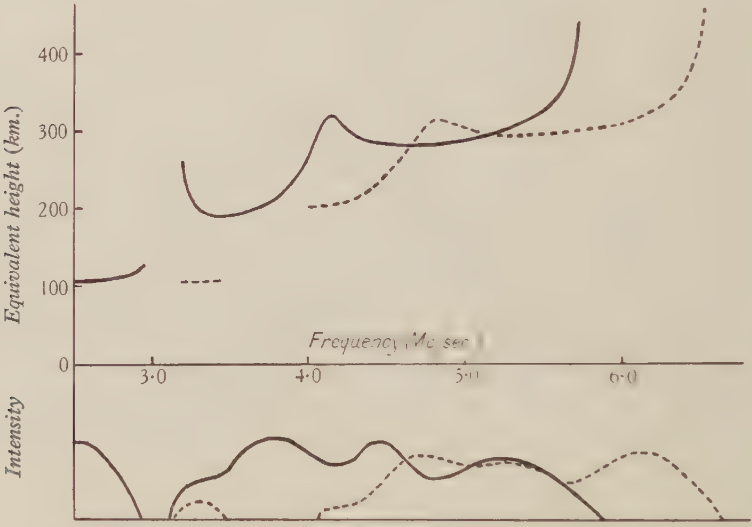

Figure 7. Ordinary ray ——, extraordinary ray - - - - -.

Appleton[14] has observed that the $\{P', f\}$ curves for the two components can b made almost to coincide by moving one of them through about 0·7 Mc. sec F. W. G. White[15] has given curves showing that the variation of the intensit with time for a fixed frequency follows the same sequence for both component and that the extraordinary ray is less intense than the ordinary ray when referre to corresponding points on the curve of equivalent height. It is seen that this hold also for the $\{P', f\}$ curves, for in figure 8 the curves of figure 7 have been redraw with the curve for the extraordinary ray displaced by $-0{\cdot}7$ Mc. sec., which make possible a comparison of corresponding points on the two curves. In general, th extraordinary ray is less intense than the ordinary ray at corresponding point though the difference is less marked at the higher frequencies, because the wave frequency is far removed from the collisional frequency of the electrons in the layer.

The results indicate, then, that there is ample evidence in favour of the magneto-ionic theory of propagation in the ionosphere, but that deductions from observations made at a single frequency may be misleading unless referred to the $\{P', f\}$ curve appropriate to the time of observation.

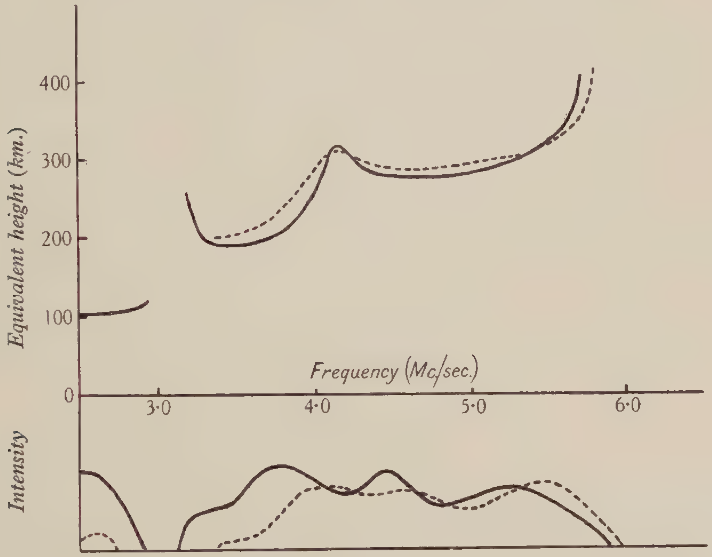

Figure 8. Ordinary ray ——, extraordinary ray - - - - -.

§ 9. ACKNOWLEDGMENTS

The work described in this paper was carried out as part of the programme of the Radio Research Board of the Department of Scientific and Industrial Research. I wish to express my indebtedness to Professor E. V. Appleton for his continued help and encouragement.

APPENDIX A

In a circuit consisting of a resistance R, a condenser C, and an inductance L in series, the current i which flows when an e.m.f. e of frequency ω is applied is given by

$$i\{R+j(L\omega-1/C\omega)\}=e,$$

and the phase angle α is given by

$$\tan\alpha=(L\omega-1/C\omega)/R.$$

Now if α is small $\tan\alpha=\alpha$

so

$$\delta\alpha=\frac{1}{RC\omega}\frac{\delta C}{C} \quad \text{or} \quad \delta C/C=\delta\alpha/Q,$$

where Q is the voltage step-up at resonance.

It is desirable to limit the error due to the difference in phase angles of the tw loop circuits to ±1 per cent and, since the error is $\pm\sin\alpha$, from equations (1 and (2), this requires that α (or in the present notation $\delta\alpha$) shall be less than 0·0 radians. The value of Q will be about 10, whence $\delta C/C=0{\cdot}001$, i.e. the capacit must be accurate to 0·1 per cent, or the condenser-setting must be accurate t 0·1 of a division. A setting of this accuracy is barely possible except with refine apparatus, so that it is not feasible to rely on a calibration of the condenser dial The rate of change of phase angle with capacity is most rapid near the resonanc point, but it is still appreciable when the amplitude of the current has fallen to on half that at resonance.

APPENDIX B

If we denote the resistances and capacitances in the bridge circuit of figure by R and C respectively, an e.m.f. e equal to $E\sin\omega t$ applied across one diagon will produce in either branch a current $e/(R-j/C\omega)$, and therefore the e.m.f. acros the other diagonal is

$$e\,(R+j/C\omega)/(R-j/C\omega),\quad \text{which}=(R^2-1/C^2\omega^2+j.2R/C\omega)/(R^2+1/C^2\omega^2)$$

and the phase angle θ is given by

$$\tan\theta=2RC\omega/(R^2C^2\omega^2-1).$$

If $R^2C^2\omega^2=1$, i.e. if $\omega=1/RC$, then $\theta=\pi/2$ and the e.m.fs. across the tw diagonals are in quadrature.

If θ is replaced by $(\pi/2-\theta)$, then $\tan\theta=(RC\omega-1)/2RC\omega$ and approximatel $\delta\theta=(RC+1/RC\omega^2)\delta\omega/2$. When $\omega=1/RC$ this reduces to $\delta\theta=\delta\omega/\omega$.

As in Appendix A, $\delta\theta$ must not exceed 0·01 radians so that the frequency the modulating oscillator must be correct and constant to 1 per cent. Under norm conditions this accuracy is easily maintained.

To determine the sense of rotation of the equivalent loop, we note that i figure 1 the e.m.f. e applied to the bridge is that across the diagonal ac and t e.m.f. across the other diagonal is that across bd. Since $\tan\theta$, above, is positiv V_{bd} leads V_{ac} and therefore one side of the equivalent loop passes through W, E, S in succession and, if we look down on the loop, this corresponds to a clockwi or right-hand sense of rotation. It will be seen that the operation of the switch reverses this sense.

APPENDIX C

To investigate the effects of magneto-ionic dispersion on the propagation of pulse of radio-frequency energy, consider an emitted wave defined by $\cos\omega t\,f\,($ where ω is the carrier pulsatance and $f(t)$ is the modulation envelope. If we assum for simplicity, a symmetrical pulse, $f(t)$ can be expanded in a Fourier cosine seri and if, further, the recurrence pulsatance is taken as unity, $f(t)$ can be written

$$f(t)=a+\sum_{n=1}^{n=a} b_n\cos nt,$$

and the pulse of radio-frequency energy becomes (if we omit the limits of summation)

$$\cos \omega t\,(a+\Sigma b_n \cos nt)$$

which
$$=a \cos \omega t+\tfrac{1}{2}\Sigma b_n \{\cos (\omega+n)\, t+\cos (\omega-n)\, t\}.$$

Now the right- and left-hand circularly polarized components of this wave, namely

$$a\,(\cos \omega t \pm j \sin \omega t)+\tfrac{1}{2}\,\Sigma b_n\, \{[\cos (\omega+n)\, t \pm j \sin (\omega+n)\, t] \\ +[\cos (\omega-n)\, t \pm j \sin (\omega-n)\, t]\},$$

travel independently in the ionosphere. Assuming that there is a difference between the paths P_r and P_l of the right and left-hand components, but that these two paths are independent of frequency, the reflected waves arriving at the receiver are

$$a\,[\cos \omega\, (t-\theta)+j \sin \omega\, (t-\theta)]+\tfrac{1}{2}\Sigma b_n\, [\cos (\omega \pm n)\, (t-\theta)+j \sin (\omega \pm n)\, (t-\theta)]$$

and

$$a\,[\cos \omega\, (t-\phi)-j \sin \omega\, (t-\phi)]+\tfrac{1}{2}\Sigma b_n\, [\cos (\omega \pm n)\, (t-\phi)-j \sin (\omega \pm n)\, (t-\phi)],$$

where $\theta=P_r/c$, $\phi=P_l/c$, and c is the velocity of light.

If these waves are received on a simple loop aerial lying in the plane

$$(\cos \alpha+j \sin \alpha),$$

the e.m.fs. induced are given by

$$a \cos \{\omega\, (t-\theta)+\alpha\}+\tfrac{1}{2}\Sigma b_n\, [\cos \{(\omega+n)\, (t-\theta)+\alpha\}+\cos \{(\omega-n)\, (t-\theta)+\alpha\}]$$

and

$$a \cos \{\omega\, (t-\phi)-\alpha\}+\tfrac{1}{2}\Sigma b_n\, [\cos \{(\omega+n)\, (t-\phi)-\alpha\}+\cos \{(\omega-n)\, (t-\phi)-\alpha\}]$$

or
$$\cos \{\omega\, (t-\theta)+\alpha\}\, [a+\Sigma b_n \cos n\, (t-\theta)]=\cos \{\omega\, (t-\theta)+\alpha\}\, f\, (t-\theta)$$

and
$$\cos \{\omega\, (t-\phi)-\alpha\}\, [a+\Sigma b_n \cos n\, (t-\phi)]=\cos \{\omega\, (t-\phi)-\alpha\}\, f\, (t-\phi).$$

Thus the envelopes of modulation are reproduced at the receiver after times θ and ϕ and these two pulses are the right and left-hand circularly polarized components predicted by theory, and they exist individually at the receiver. When combined in the receiver they lose their identities, and because of the vector summation the resultant pulse bears little resemblance to the original pulses, unless they are completely resolved.*

Instead of finding the separate effects of the two circularly polarized components, Handel and Plendl combine them into a plane-polarized wave, represented by

$$2a \cos \omega \left(t-\frac{\theta+\phi}{2}\right) \left[\cos \omega\, \frac{\theta-\phi}{2}-j \sin \omega\, \frac{\theta-\phi}{2}\right] \\ +\Sigma b_n \cos (\omega \pm n) \left(t-\frac{\theta+\phi}{2}\right) \left[\cos (\omega \pm n) \left(\frac{\theta-\phi}{2}\right)-j \sin (\omega \pm n) \left(\frac{\theta-\phi}{2}\right)\right]$$

---

* The conditions which have been assumed are very unusual in practice but it can be shown(16) that, in the presence of frequency dispersion, expressions similar to the above are derivable, though the phase of the carrier wave is determined by the optical path, whereas the envelope of modulation is reproduced after the equivalent path of the atmospheric waves; the shape of the modulation envelope is not appreciably affected, except under certain extreme conditions.

from which it is apparent that the plane of polarization varies with the side-banc frequency. The e.m.f. induced in the aerial becomes

$$2a \cos \omega \left(t-\frac{\theta+\phi}{2}\right) \cos \left\{\omega \left(\frac{\theta-\phi}{2}\right)+\alpha\right\}$$

$$+\Sigma b_n \cos (\omega \pm n) \left(t-\frac{\theta+\phi}{2}\right) \cos \left\{(\omega \pm n) \left(\frac{\theta-\phi}{2}\right)+\alpha\right\}.$$

The summation of the equivalent of this expression is carried out graphicall by the authors mentioned, and the possibility of resolving the resultant comple pulse into two overlapping pulses is obscured since the two amplitudes at an point have to be added vectorially. It should also be noted that the polarizatio varies throughout the overlapping portion, though the beginning and end ar circularly polarized with opposite senses of rotation.

The two methods of attack must be strictly equivalent, however, and the abov expression readily reduces to

$$\cos \{\omega (t-\theta)+\alpha\} f (t-\theta)+\cos \{\omega (t-\phi)-\alpha\} f (t-\phi),$$

as was found previously.

In determining the time delay of the components of a complex echo, it is useles to take the points of zero amplitude. The echo must first be resolved into two pulse similar in shape to the emitted pulse, and the beginning of these two pulses wil give the correct delay for the two components. If measurements can be mad from the end as well as the beginning of the pulses, this procedure is not necessar

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MEASUREMENTS OF THE THERMAL EXPANSION OF CAST AND ROLLED ZINC

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ABSTRACT. The coefficients of expansion of cast and rolled zinc are measured under conditions of steady and of continuously increasing temperature, and marked differences between the coefficients are found to occur. On rolling, the random orientation of crystals in the cast specimen is broken up and most of the long axes are set perpendicular to the plane of rolling. No change is produced in the volume coefficient by rolling. When the zinc crystals are small, the continuous expansion is also small but as the size of the crystals increases, the continuous expansion also increases. Discontinuities in the continuous expansion are traced to the long axis of the crystals.

§ 1. INTRODUCTION

This paper deals with the expansion of small specimens of cast and rolled zinc, as measured with steady and with continuously increasing temperatures. In the latter method, the specimen is heated at a steady rate and simultaneous readings of length and temperature are taken during the heating; observations are also taken during cooling. An optical-lever method measuring down to $8\cdot2 \times 10^{-6}$ cm. is used to measure the changes of length, while temperatures are measured with a nickel-nichrome thermocouple reading to 0·7° C. The specimens are 0·16 cm. high and can be heated to 405° C. with safety.

The scope of the investigation can best be seen from the following summary. (i) The expansion of the zinc is compared with that of glass, whose expansion is compared with that of silica. The glass has a constant coefficient of $8\cdot6 \times 10^{-6}$ up to 240° C. During cooling a slightly lower coefficient is obtained. (ii) With steady temperatures, well annealed cast zinc gives a coefficient of $30\cdot2 \times 10^{-6}$ up to 250° C.; this shows that a random orientation of crystals exists. (iii) Measurements on cast zinc at continuously increasing temperatures reveal sharp discontinuities at about 200° C. in the slope of the {expansion, temperature} graph. Heating the specimen to 400° C. before the experiment moves the discontinuity to a lower temperature around 130° C. After quenching of cast zinc, a slight negative movement is obtained at lower temperatures followed by a slight expansion. On annealing, the expansion increases with the amount of annealing. Thus in the small specimens used, the expansion with continuously increasing temperatures is influenced by the state and growth of the zinc crystals. (iv) The continuous-cooling curves give reliable information and from them the coefficients of expansion of annealed and unannealed zinc can be compared. (v) Steady-temperature measurements with rolled zinc 0·03 cm. thick show that 90 per cent of the short axes of zinc crystals are in the direction of rolling and 80 per cent of the short axes are in the plane but perpendicular to the direction of rolling. (vi) Continuously increasing temperatures with the rolled zinc show that the discontinuities in the expansion are associated with the long axes of the crystals. (vii) The volume coefficient of the zinc is found to be unaltered by rolling.

§ 2. APPARATUS

The inverted form of furnace and optical system used have already been described$^{(1)}$ and it will be sufficient to describe the mounting of the specimen figure 1. A tripod *D* rests with its outer legs on glass and its centre leg on the specimen. A microscope cover 0·01 in. thick was broken into three pieces and a piece was placed under each foot of the tripod to prevent the foot from sinking into the

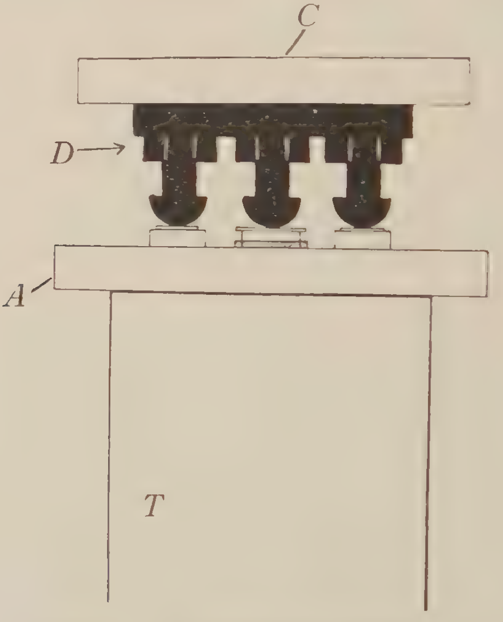

Figure 1. The mounting of the specimen.

zinc at higher temperatures. Light passes up the silica tube *T* through the glas window *A* and is reflected by the mirror *C* back approximately along its own path

The legs of the tripod are made as nearly equal as they could be made b adjustment with vernier calipers, and then an expansion experiment is performe without any specimens. A very slight adjustment of one of the legs is sufficien to eliminate the expansion of the tripod.

Figure 1 shows a rolled specimen 0·030 cm. thick under the centre leg of th tripod. The shaded strip under the centre leg is a small piece of commercial zin in which are cut two slits; two pieces of rolled zinc are pushed into the slits and s held on edge. The microscope cover piece is now placed across the vertical zincs and the foot of the tripod rests on the glass cover. Strangely enough this gives an exceptionally steady system that is not influenced by outside disturbances.

In calculating the coefficient $(I/L)\,dL/dT$ for continuously increasing temperatures, a minimum overall accuracy of 1 in 50 has been maintained, but for steady temperatures a much greater accuracy is reached, the expansion being measured with an error never greater than 1 in 150, while corresponding temperatures are measured to 1 in 160. The coefficient obtained is in turn averaged out on the {coefficient, temperature} curve.

§ 3. RESULTS FOR GLASS

Focusing difficulties prevented zinc from being compared directly with silica, so the expansion of the zinc was compared with that of glass which in turn was compared with that of fused transparent silica. The coefficient of expansion of silica is quoted as $0{\cdot}54 \times 10^{-6}$ up to 1000° C.

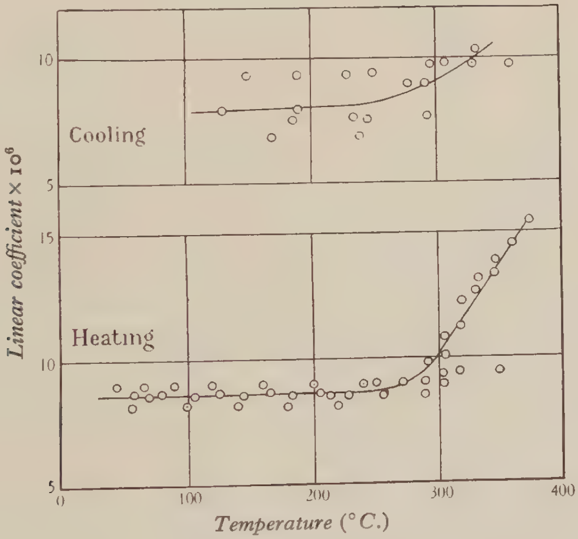

Figure 2. The coefficient of expansion of glass.

After the glass in the furnace had been heated a number of times, observations were taken, and figure 2 gives the coefficient obtained on four occasions with continuously increasing readings of expansion and temperature. This coefficient is actually $(I/L)\,dL/dT$, i.e. the rate of change of length with temperature at a particular temperature. Excellent agreement is obtained for the heating curve, but the cooling curve shows fatigue, as might be expected. At a later date the glass was again compared with silica, the temperature and expansion being kept constant for at least five minutes before a reading was taken. The coefficient thus obtained for steady temperature is identical with the heating coefficient shown in figure 2 for continuously increasing temperatures. Table 1 gives the heating coefficien $(1/L)\,dL/dT$ for glass.

Table 1

Temperature (°C.)	$\frac{1}{L}\frac{dL}{dT}\times 10^6$
30	8·55
200	8·63
300	10·10
380	16

Various rates of heating, such that from 38 to 64 minutes were taken in heatin through 310° C., have been used in the continuously increasing temperatur measurements, but the coefficient is independent of the rate of heating. The glas was tested at frequent intervals and it is quite certain that the anomalous effect described later are not due to the glass.

§ 4. RESULTS FOR CAST ZINC

To measure the expansion of the cast zinc under conditions of steady tem perature, a specimen 0·1780 cm. thick was annealed inside the furnace at 350° C for 56 hours. The specimen was allowed to cool slowly inside the furnace, and it expansion was then compared with that of the glass. The length of the zinc in creased up to 403° C., the highest temperature to which it was heated. Figure gives the corrected coefficient $(1/L)\,dL/dT$ for the cast zinc and on the sam

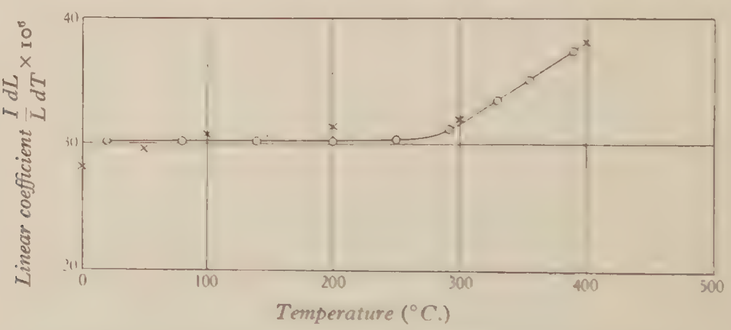

Figure 3. Annealed zinc; × Owen and Yates; ⊙ present work.

figure is shown the coefficient worked from the results of Owen and Yates In using the results of Owen and Yates, quoted for the axis of the zinc crystals the average linear coefficient for the zinc is taken as $\frac{1}{3}$ (coefficient long axis $+ 2 \times$ coefficient short axis). Exceptionally good agreement is obtained in the result including the upward trend above 260° C. The agreement indicates the reliabilit of the method and also indicates a random orientation of zinc crystals inside a we annealed specimen of cast zinc.

Table 2 gives the mean coefficient over the range extending from 20° C. t

100° C. as derived from the present measurements and from the results of Gruneisen and Goens(3).

Table 2

Mean coefficient for cast zinc from 20° C. to 100° C. × 10^6		
Present measurements	Owen and Yates	Gruneisen and Goens
30·3 ± 0·2	29·8	30·7

§ 5. CONTINUOUSLY INCREASING TEMPERATURES

Before the expansion of the cast zinc was examined under conditions of steady temperature, a specimen was examined under continuously increasing temperature. Sharp changes occur in the slope of the {expansion, temperature} curves; so sharp indeed are these changes that they constitute well marked discontinuities. Table 3 indicates their main aspect. From the table it will be seen that heating the specimen immediately before the test has the effect of moving the discontinuity to a lower temperature, while continued heating causes the discontinuity to disappear. It has been found that these discontinuities do not depend on the rate of heating or on changes in the rate of heating.

Table 3

Curve no.	Date	Whether previously heated	Observed heating	Discontinuity at ° C.	Coefficient up to discontinuity × 10^6
V	May 16	—	3	210	29·1
VI	„ 19	—	4	192	29·6
VII	„ 22	Yes	6	105	24·3
VIII	„ 23	Yes	8	155	25·7
IX	June 13	—	15	—	22·6
X	„ 26	—	16	—	24·3

§ 6. HYPOTHESIS

It is suggested that using very small specimens gives rise to an anomalous effect that would not be detected in large specimens, and that the growth of zinc crystals during a heating experiment may affect the apparent expansion of the zinc. The following experiments have been carried out to test this hypothesis. A specimen was kept inside the furnace at 350° C. for 20 minutes; the top of the furnace was quickly removed and the specimen was quenched in tap water. On being quenched the crystals are subjected to severe strain and will be broken up into smaller crystals. The day after quenching, the expansion of the specimen was compared with that of glass by the continuously-increasing-temperature method. The result is shown in figure 4 as curve XXXI. The movement in the eyepiece is exceptionally small and in addition gives a small negative expansion at lower temperatures. This negative expansion is similar to that obtained at lower temperatures for the first direction (see later) of the rolled zinc. Hence the first direction of the rolled zinc—the direction perpendicular to the plane of rolling—contains a large percentage of small crystals.

The following day, a second experiment was performed on the same specimen without further annealing. The result is shown as curve XXXII, figure 4. These two curves would not be expected to be identical, because the specimen has been annealed to some extent during its first expansion experiment. The general features of the first curve, including the negative movement at lower temperatures, are however, reproduced in curve XXXII. The linear coefficient, corrected for glass over most of the heating range is 10×10^{-6}.

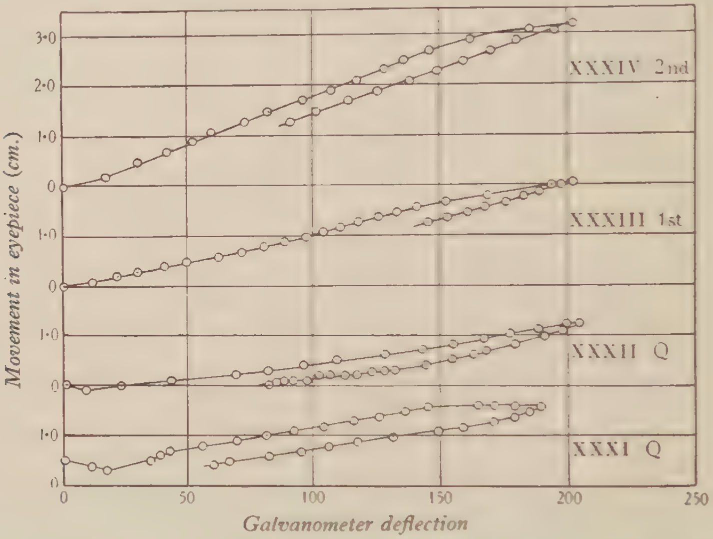

Figure 4. Cast zinc: continuously-increasing-temperature curves obtained after varying heat treatment.

If the specimen is now annealed at a fairly high temperature, some of the crystals will be given a chance to grow at the expense of others and the expansion curve should, on this hypothesis, alter. Accordingly, the specimen was placed inside the furnace without the tripod, mirror, etc., and kept at a temperature of 350° C. for 14 hours. This time does not include the time (about 40 minutes) taken to heat the furnace or the time taken for cooling. During cooling, the 1-inch observation hole in the bottom of the furnace was plugged with asbestos, otherwise strain might be set up in the crystals causing them to be broken once more.

The specimen was subjected to increasing amounts of annealing at 350° C., the expansion being measured between each annealing. Some of the experimental curves are shown in figure 4, while table 4 gives the data. Curve XXXVII is the steady-temperature curve which has already been quoted in the results for cast zinc.

The coefficient $(1/L)\,dL/dT$, corrected for glass, for all the above curves is shown in table 4. This table shows clearly that on quenching the small crystals are in a state that retards expansion. The coefficient of expansion increases with the

Table 4

Curve no.	Date	Type	Total hours annealing	Heating current	Coefficient $\times 10^6$ at 150° C.	Symbol in figure 5
XXXI	Jan. 24	After quenching	—	1·2	17·5	⊙
XXXII	,, 25	After quenching	—	1·2	15	⊡
XXXIII	,, 31	After first annealing	14	0·6 to 1·6	20·4	×
XXXIV	Feb. 7	After second annealing	26	1·2	31·1	⊠
XXXV	,, 15	After third annealing	43	1·0	35·5	⊞
XXXVI	,, 21	After fourth annealing	56	1·4	41·2	⊕
XXXVII	,, 22	Steady	56	—	31·2	

first and second annealings, when the crystals grow, and approaches the coefficient relating to steady temperature. After the third and fourth annealings, the coefficient of expansion exceeds that related to the steady state and it is suggested that these curves correspond to the actual growth of crystals. This view is supported by the way in which the fourth curve falls to the steady-temperature curve at 220° C., as if a saturation point had been reached.

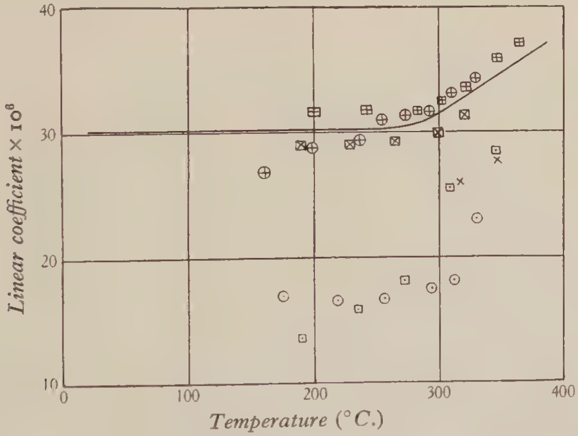

Figure 5. Cast zinc: cooling values during treatment. The continuous line is the steady heating curve.

Twenty days after the steady-temperature curve had been obtained, another continuous-expansion experiment was performed on the same specimen without further annealing. Up to 152° C., the expansion was similar to that after the first annealing; above 152° C., the expansion was similar to that after the fourth annealing, the discontinuity in this curve being identical in nature with the discontinuities already mentioned. On cooling, the coefficient was larger than that obtained in an experiment immediately preceded by annealing.

Figure 5 shows all the cooling-coefficients for the heat-treatment tests of table 4.

The outstanding point of this figure is the exceptionally good agreement obtained between the coefficients for the different amounts of annealing, except in the cases of the quench and first annealing. The full line through these points is the heating-coefficient found with steady temperatures; hence the cooling-coefficient with continuously changing temperature gives reliable information for the cast zinc provided the specimen has been annealed for at least 26 hours.

These results indicate that the continuous expansion of cast zinc containing very small crystals is small also, but as the crystals grow the expansion increases. If the specimen is not annealed before the experiment, the expansion is influenced by the growth of crystals during the experiment.

§ 7. ROLLED ZINC AT STEADY TEMPERATURES

A cast zinc specimen 0·4 cm. thick was rolled down to 0·030 cm., the same end being inserted between the rollers for each rolling. Between each rolling the specimen was heated on the forge, allowed to cool for a few seconds, and then gently rolled. There was sufficient oil on the rollers to quench the small specimen used. An interval of two months was allowed to elapse before expansion experiments were performed on the rolled zinc, so that any periodic fluctuations set up by the working would have time to disappear. The expansion of the rolled zinc was examined in three directions at right angles to each other called the *first, second, and third directions*. .

The first direction is perpendicular to the plane of rolling, the second is in the plane and in the direction of rolling, i.e. along the length of the rolled strip; and the third is in the plane but perpendicular to the direction of rolling, i.e. across the rolled strip.

First direction. To obtain the expansion for the first direction, small pieces of the rolled zinc were made into a pile of five specimens 0·1501 cm. thick. The expansion of this pile was compared with the expansion of the same pieces of glass that were used for the cast zinc.

Before the expansion at steady temperatures was measured the specimens were heated a number of times and, in addition, were annealed at 350° C. for 22 hours. The expansion was measured on the three days following annealing. Curve XXXX, figure 6, gives the results. At lower temperatures the expansion was slight but negative, and irregular in that the results obtained on different occasions did not agree, and also irregular in that lagging took place. Around 100° C. the coefficient became positive and from 200° C. to 300° C. fair agreement was obtained between five readings.

In figure 7 the coefficient $(1/L)\,dL/dT$ is shown for this work and also for the long and short axes of the zinc crystals from the results of Owen and Yates[(2)]. The agreement indicates that by the rolling of the specimen down to 0·030 cm. a large percentage of the zinc crystals have been turned, so that their long axes are now perpendicular to the plane of rolling. At lower temperatures the expansion is irregular, the irregularity probably being due to the fact that the crystals in this direction are very small (see the results for continuously changing temperature, quoted later).

Second and third directions. The expansion in the second direction measured with steady temperatures is shown in figure 6, and a portion of the cooling curve also is shown. The highest reading on this curve was kept steady for 37 minutes. Up to 240° C. the coefficient is constant at $20{\cdot}5 \times 10^{-6}$, as shown in figure 7. Also in figure 7 will be seen the coefficient $(I/L)\, dL/dT$ worked out for the short axis of the zinc crystals from the results of Owen and Yates(2). The coefficient for the second direction lies close to the coefficient for the short axis, and if the two temperatures of 130° C. and 170° C. (which appear from the graph to be reasonable or average temperatures) are taken for computing the percentage of short axes in the

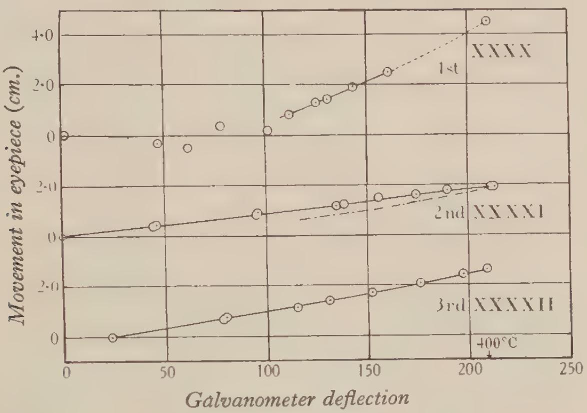

Fig. 6. Rolled zinc compared with glass. Steady temperatures.

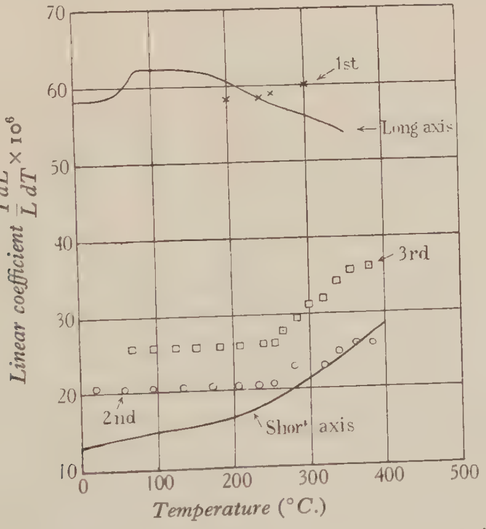

Fig. 7. Coefficients of rolled zinc and long and short axes of zinc crystals.

second direction, then the percentage of short axes is 89 at 130° C. and 91 at 170° C. Thus, when zinc is rolled down to 0·030 cm., 90 per cent of the short axes of the crystals lie in the direction of rolling.

The steady-temperature expansion for the third direction also is shown in figure 6 and the coefficient in figure 7. The coefficient for the third direction lies between the coefficients for the long and short axes of the zinc crystal and, an average being taken as above, it may be concluded that when the specimen is rolled down to 0·030 cm., 80 per cent of the short axes lie in the plane of rolling but perpendicular to the direction of rolling.

§ 8. ROLLED ZINC AT CONTINUOUSLY CHANGING TEMPERATURES

The expansion of the rolled zinc has also been measured with continuously changing temperatures, the specimen being mounted in the same manner as for the steady temperatures. Curve XIX, figure 8, gives a typical continuous-expansion curve for the first direction. Even after the expansion of glass has been allowed for the curve, XIXA, still indicates a slight negative movement at lower temperatures. The minimum on this curve has been obtained on eleven occasions, all with steady heating-current ranging from 0·6 to 1·5 amperes, and in each case the temperature of the minimum has been close to 87° C. During this work the specimens were piled in a different order, allowed to stand for some days, heated to 400° C. and allowed to cool slowly before an expansion experiment, but still the minimum occurred. Now a minimum similar to this occurs in cast zinc immediately after quenching when the zinc crystals are small; hence it is inferred that after rolling also, the crystals are small. Since this minimum is only found in rolled zinc for the first direction, the suggestion is put forward that it is the long axes of the zinc crystals that are associated with the discontinuities obtained for the cast zinc. This

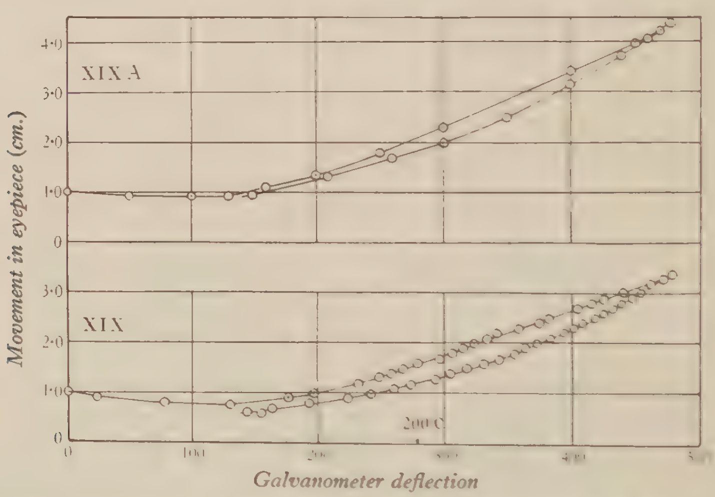

Fig. 8. Typical of 1st direction of rolled zinc. Top curve is corrected for the expansion of glass.

view is supported by the fact that very little difference is found between the heating and cooling coefficients for the second direction containing 90 per cent of short axes.

The third direction of the rolled zinc contains 20 per cent of long axes, and when it is heated bumps are obtained in some three of the continuous-expansion curves. However, the next two heatings of the specimens, without annealing, did not show the bumps but gave constant heating-coefficients of 24·3 and $25{\cdot}1 \times 10^{-6}$. The cooling-coefficients for all these five curves are in excellent agreement with each other.

In the tests of heat treatment for cast zinc, it was found that the cooling curves gave reliable information. Hence by means of the cooling curves an average coefficient for the rolled zinc can be worked out at any temperature by adding the coefficients for the three directions and dividing by 3. Figure 9 shows the average coefficient for the rolled zinc found in this way.

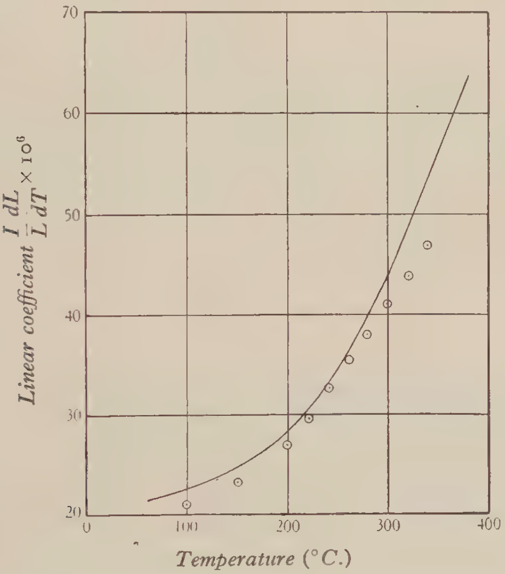

Figure 9. Unannealed zinc: —— cast; ⊙ rolled.

Figure 9 also shows the average coefficient obtained with continuous temperatures for the three directions of the unannealed cast zinc, and this is practically a cooling-curve. The agreement between these two curves for continuous temperatures is too good to be accidental even though they are both very different from the steady-temperature curve of figure 5. It is suggested that the continuously-changing temperature curves are valid for unannealed zinc while the steady-temperature curve applies to annealed zinc. It follows from figure 9 that the volume coefficient of the zinc remains unaltered after the specimen has been rolled down to 0·030 cm.

The expansion of a metal is usually represented by the equation

$$L = L_0\,(1 + \alpha T + \beta T^2 + \gamma T^3 + \delta T^4),$$

where α, β, γ and δ are constants for the metal. On differentiation

$$\frac{1}{L}\frac{dL}{dT}=\alpha+2\beta T+3\gamma T^2+4\delta T^3.$$

This last expression is what has been called the "coefficient" in this paper. Hence the values of the constants can be found.

	$\alpha\times 10^6$	$\beta\times 10^9$	$\gamma\times 10^{12}$	$\delta\times 10^{15}$
Cast zinc unannealed (author)	27·84	−47·52	178	− 18
Cast zinc annealed (author)	29·65	10·84	− 68·4	129
Annealed zinc (Owen and Yates)	28·04	23·39	− 81·11	118·2

§ 9. ACKNOWLEDGMENTS

I am extremely grateful to Prof. E. A. Owen of the University College of North Wales, Bangor, for constant advice and encouragement, without which the work could not have been carried out. My thanks are also due to the Headmaster of the County School, Rhyl, for placing at my disposal the facilities of the school laboratories, where the work was done.

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OBITUARY NOTICES

FRANCIS JOHN WITTS

Mr Francis John Witts was born at Newport, Monmouthshire in September 1878. He was apprenticed to Messrs Dunscombe and Co., opticians, of Bristol. He came to London in 1908, and in the same year was elected a member of the Optical Society. In 1914 he opened a business as an ophthalmic optician in the Buckingham Palace Road, and was particularly successful in dealing with patients requiring difficult prescriptions. He was an ingenious man, and invented a rimless pince-nez fitting in which the metal work or clip was made in one piece. He specialized in the drawing of fine wires, and in 1925 invented* a form of double jewel draw-plate with a view to improving the quality of the wires drawn. Mr Witts had a kind and generous disposition. He died on March 4, 1934.

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* British patent specification No. 257,366.

C. F. B. KEMP, A.R.C.S., B.Sc., D.I.C.

The death of Charles Frederick Barnard Kemp in a motor accident on July 21, 1934 at the very early age of 31 years has terminated a career which promised to be very successful.

Mr Kemp entered the Royal College of Science in 1921 and took the associateship course in physics. He was awarded the A.R.C.S. (1st Class) in 1924, and took his B.Sc., London, degree (honours physics) in the following year. He was appointed to the Research Department, Woolwich Arsenal in 1925, and transferred to the A.D.E.E. in 1929, where he remained until his untimely death.

During the last few years he had undertaken pioneer work on the analysis of aeroplane noise, and he attacked this most difficult problem with great energy and resource. Some of the experimental results have appeared in the two contributions which he made to the *Proceedings of the Physical Society*, and one has only to glance through these papers to realize what pains he had taken to master the difficult technique of outdoor sound-intensity measurements.

He had been a member of the Physical Society, as Student and Fellow, since 1921.

SIR ARTHUR SCHUSTER, F.R.S.

ARTHUR SCHUSTER was born at Frankfurt-on-Main on September 12, 1851 His family was of Jewish origin and had been settled in Frankfurt for a century or more when, in the middle of the eighteenth century, the firm of Gebrüder Schuster trading in English cotton and other goods was founded. When Napoleon after the conquest of Prussia in 1806 confiscated the goods of the firm as English several members of the family moved to England, and Manchester became the firm's headquarters, with Frankfurt, London and Bradford as branches. In several of them banking business grew up automatically, and by the middle of the century the Frankfurt business was almost exclusively banking, with Arthur Schuster's father Francis Joseph (born 1823) as head. He had married in 1849 Marie (born 1830), daughter of Hofrat Pfeiffer, one of the founders of the Hofbank at Stuttgart By this time many members of the family, including all those in England, were Christian, and Francis and Marie became Christian early in their married life and their children were brought up in that faith. She was musical and as a membe of the local choral society took part in its sacred concerts. Arthur, the second son began school at 5 and entered the Gymnasium at 12. Although from his schoo reports he appears to have made satisfactory progress, he was far from happy in his school life and left at 16½ without regret. In the autumn of 1868 he was sent as his elder brother Ernest* had been the year before and as his younger Felix† was the year after, to live with the Protestant Pastor Goetz at Geneva. Here he studied French, and during his two years' stay attended lectures in history, commercial geography, mechanics, physics, chemistry and anatomy at the Academy, and learn the routine work of the Observatory. He regarded this as one of the happies periods of his life, and left Geneva with regret in May 1870 for Manchester, where his father had the previous year replaced in the business an uncle who had died In due course the family was naturalized, and in 1873 Francis Schuster moved to London to join the firm of Schuster, Son and Co., merchants and bankers, Cannon Street, which had been founded in 1845, and part of whose business was taken ove by the Union Bank of London in 1887.

In the autumn of 1870 Arthur commenced work in the Manchester office o his father's firm. He had, however, set his heart on a scientific career, and he began to attend the evening lectures on Chemistry given by Prof. Roscoe in the Owens College. At the end of the course, in April 1871, he was awarded the first prize and his mother who, although then blind, had discovered how unhappy he was in office work, induced his father to consult Prof. Roscoe as to the feasibility of Arthur taking up a scientific career. As a result he entered the day classes at the Owens College in October, to study mathematics under Prof. Barker and physics under Prof. Balfour Stewart, and to commence spectroscopic research under Prof. Roscoe At the end of the session he was awarded the first prizes in the mathematics natural philosophy and practical physics classes, and the second prize in the extra physics class. His father was quite satisfied as to his career and took great pride in all his future work. In the summer he communicated some of the results of his research to the British Association at Brighton, and to the Royal Society. The winter he spent at Heidelberg under Professors Kirchhoff, Bunsen and Königsberger and in the summer of 1873 the degree of Ph.D. of that University was conferred on him. In the autumn he was appointed honorary demonstrator in the Physics Department of the Owens College, and commenced a research on the validity of Ohm's law with Prof. Stewart in the new college buildings in Oxford Street. In 1874 at the close of the session he spent a few weeks in Prof. Weber's primitive laboratory at Göttingen, the autumn in Prof. Helmholtz's laboratory at Berlin, and Christmas at his father's house at Hampstead. Early in 1875 he was asked to join and ultimately to take charge of the Royal Society expedition to Siam to observe the solar eclipse of April 6. On the return journey he spent two months in a tour across the Himalayas into Kashmir, and in November he resumed the demonstratorship and his optical work at the Owens College, and lectured to advanced students on Maxwell's "Electricity and Magnetism" which had appeared two years previously.

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* Afterwards K.C. and an authority on international law.

† Afterwards Sir Felix Schuster, Bart., of the Union of London and Smiths Bank.

In May 1876 he visited Maxwell at Cambridge with a view to beginning measurements of the magnetic properties of rocks, but after discussing with him the apparatus which would be required he decided to continue his spectroscopic work. He entered the Cavendish Laboratory in October and was admitted a fellow commoner of St John's College in December. In 1878 he went to Colorado to take observations during the solar eclipse of July 29, and in 1879 was elected a Fellow of the Royal Society. When after the death of Maxwell Lord Rayleigh became Professor at the beginning of the Lent term, 1880, Schuster joined with him in a re-determination of the British Association ohm, the value of which was then uncertain to about 4 per cent. In the following Michaelmas term he lectured on radiation to post-graduate students. In 1881 he was elected to the newly constituted chair of applied mathematics at the Owens College, which had become one of the constituent colleges of the Victoria University. He there commenced the researches on the passage of electricity through gases which he described in his Bakerian Lectures to the Royal Society in 1884 and 1890. In 1882 he went with Abney to Egypt to photograph the corona during the solar eclipse of May 17, and in 1886 with L. Darwin and Maunder to the West Indies for that of August 29.

In 1887 he married Cary, the eldest daughter of George Loveday of Wardington, Banbury, Oxon., and in 1888 succeeded Balfour Stewart as Professor of Physics at Manchester.

The great increase in the number of students taking physics necessitated the addition of three rooms to the three which had served previously as the physical laboratories, and in 1898 an independent building for physics, to the design of which he gave much time and thought, was commenced. It was opened in 1900.

When the Charter of the University of Manchester was granted in 1903 Schuster became Dean of the Faculty of Science and representative of the University on the Education Committee of Manchester. His father died in 1906, and he resigned his

Professorship in 1907, being succeeded by Rutherford. Already a good painter in water colours, he now took up oil painting as a recreation and in the course of a tour of India in 1908 he produced a number of oil paintings of Indian scenery.

From 1912 to 1914 he was President of the Physical Society. From 1912 to 1919 he was Secretary and from 1920 to 1924 Foreign Secretary of the Royal Society. In 1913 he removed from Manchester to Twyford, Berks. He was on his way to the Crimea for the solar eclipse of August 21, 1914 when the great war broke out and he only succeeded in returning from Constantinople via Egypt at the end of September. Next year he was President of the British Association at the meeting in Manchester, and on the formation of the International Research Council in 1919 he was elected Secretary.

He was knighted in 1920 and in 1924 served on the Royal Commission on the Universities of Oxford and Cambridge.

In 1923 he lost the sight of one eye through an accident. It was a considerable shock to his general health and well-being, and he had to abandon his recreation of painting. A severe illness in the winter of 1927–8 left him much weakened, and he became a greater invalid as the years progressed. He died at his home at Twyford on October 14, 1934, and a memorial service was held in St James's Church, Piccadilly on October 18.

Lady Schuster, one son and four daughters survive him.

When Schuster became a student under Roscoe he was already searching for harmonic relations amongst the spectral lines of substances, and it was not till ten years afterwards that he showed that such relations were only accidental.

When 25 years later Rydberg had divided the lines of a spectrum into series, known respectively as the P, S and D series, Schuster discovered independently of Rydberg the equality of the wave-number of the first line of the P series and the difference between the limits of the S and D series, a relation which is now included in the modern theory of series spectra.

On the advice of Roscoe he commenced in the Physical Laboratory an experimental investigation into the origin of the second or band spectra of gases under low pressure, and by 1872 had concluded that the band spectrum of hydrogen was due to hydrocarbon impurities derived from the lubrication of the taps of the containing tubes and that of nitrogen was due to the oxides of nitrogen. Widening of the lines of the spectra of gases he ascribed a year later to increase of pressure exclusively, and not to increase of temperature. During his tenure of the physics demonstratorship under Balfour Stewart his experimental tests of Ohm's law led to doubts as to its validity. In 1876 he solved the question of the origin of the motion of the vanes in a radiometer exposed to radiation by surrounding them with a transparent bell, which he showed rotated in the opposite direction to the vanes, thus proving that both motions were due to the action of the residual gas. At Cambridge he continued his investigations of the spectra of metalloids and was led to consider the wider question of the mechanism of the passage of electricity through gases. With Lord Rayleigh he determined the British Association standard ohm in absolute units by the rotating-coil method, and found it 1·1 per cent too small. For several years after his return to Manchester his principal experimental work was on the electrical conductivity of gases, which he held to be ionic in character. He showed that a solid obstacle in the cathode dark space of a vacuum tube cast shadows on both cathode and anode, and on deflecting the rays from the cathode by means of a magnetic field he concluded that these rays consisted of particles of nitrogen. The potential at a distance x from the cathode he expressed as $V(1-e^{-kx})$, where k was a constant. His Bakerian lectures of 1884 and 1890 dealt with this pioneering work, which led him on to the investigation of various types of disruptive discharges in gases, such as lightning, and in 1900 with Hemsalech he showed that when a spark passes through a gas the current is in the first instance carried by the gaseous ions and immediately afterwards by those of the vapourized metal of the electrodes. In 1890 he showed that the experimental work of others on the sparking-distance between electrodes of various forms could be coordinated by calculating the maximum electric field between the electrodes in each case, and in 1904 he determined the coefficient of recombination of the ions in atmospheric air as $3–4 \times 10^{-6}$ cm.^{3}/sec.

The four solar eclipses in which he had taken observations of the corona led him to consider the question of the constitution of the sun, and its effect on the electrical properties of the earth's atmosphere and the magnetic state of the earth itself. He was one of the first to work out quantitatively the consequences of Balfour Stewart's hypothesis of the existence of a conducting layer in the atmosphere, and he showed in 1889, on the assumption that the forces which produced the daily variation of the magnetic state of the earth had a potential, that those forces resided in the atmosphere and produced electric currents whose effect was modified only slightly by their inductive action on the material below the surface.

In his presidential address to the Physical Society in 1912 he reviewed some of the theories which had been put forward in explanation of the magnetism of the earth and rejected such as were based on electric currents within the surface.

Out of his researches in terrestrial magnetism his general method of searching for hidden periodicities in recorded phenomena, described in his "Periodogram" paper of 1906, appears to have developed.

His early spectroscopic work and his contact with Lord Rayleigh engendered an interest in optical questions which persisted throughout his life. In 1870 his method of adjusting the collimator of a spectroscope appeared, and in 1905 his conclusions as to the best width of slit to use. His simplified methods of dealing with diffraction and interference effects, some based on the hypothesis that light consisted of irregular impulses which were converted into periodic disturbances by the optical apparatus, appeared in a series of papers from 1891 onwards. Absorption and scattering of light (1920) and their bearing on cosmical effects (1905) also interested him.

In other fields he made determinations of the mechanical equivalent of heat, with Gannon, as 4·193 (1895), of electrochemical equivalents, with Crossley (1891 and 1894), of defects of colour vision (1890), and he wrote a series of "electrical notes".

His published works are as follows: 1896, *Intermediate Practical Physics* (with Lees); 1901, *Exercises in Practical Physics* (with Lees); 1904, *An Introduction to the Theory of Optics*; 1911, *The Progress of Physics*, 1875–1908; 1917, *Britain's Heritage of Science* (with Shipley); 1931, *Biographical Fragments*.

The Royal Society awarded him a Royal Medal in 1893, a Rumford Medal in 1926 and a Copley Medal in 1931. He was elected an honorary or a corresponding member of many scientific societies at home and abroad and his fluency in French and German made him invaluable in international conferences. He took a prominent part in the foundation of the International Association of Academies and from 1919 to 1928 was Secretary of the International Research Council. Many foreign delegates to scientific conferences in this country look back with pleasure to his hospitality.

His wide interests in science and his sound common sense rendered him a most desirable member of research committees, and he served for many years on such committees of the Royal Society and the British Association. He had terms of office as Chairman of the Executive Committee of the National Physical Laboratory and as Vice-Chairman of the Meteorological Committee of the Air Ministry. As a Secretary of the Royal Society he took an active part in the organization of scientific research required by government departments during the great war.

He gave much time and thought to the affairs of the scientific and educational institutions with which he was connected and contributed most generously to their finances.

C. H. L.

THOMAS FRANCIS CONNOLLY

Mr Thomas Francis Connolly, who died on November 23, 1934, was born in County Monaghan, Ireland, in 1878. He graduated as M.Sc. at Liverpool in 1899. He spent eight years at the Solar Physics Observatory, Kensington, at the end of which time the Observatory was transferred to Cambridge. Mr Connolly was then appointed Assistant Inspector of Scientific Supplies at the India Store Department. In 1921 he was placed in charge as Inspector of Scientific Supplies. During the war he worked at the Ministry of Munitions in connection with the supply of optical instruments, and was awarded the M.B.E. for his services.

Mr Connolly had a wide range of knowledge, and the secret of his success as an inspector was his capacity for knowing what were the essential parts in the construction of an instrument, and insisting on good workmanship in them. He invented a simple form of theodolite, a reversible level, and a portable standard compass, all three instruments being known by his name. He also published various papers in technical journals dealing with instrument problems.

SIR ALFRED EWING, K.C.B., F.R.S.,

MARCH 27, 1855—JANUARY 7, 1935

MANY appreciations of the work and personality of Sir Alfred Ewing have appeared in the Press* and to these the reader may be referred who would know something of Sir Alfred as engineer, and something of those physical researches which added so much to our knowledge of the classical magnetism, and gave a new word to the English language. There, too, the reader will find the discreet story of Room 40, and accounts of that genius for administration which revolutionized naval education and guided the destinies of the University of Edinburgh through a critical period of expansion.

Very scant reference, however, has been made to Sir Alfred's work in furthering the study of the scientific basis of refrigeration, and the writer of this note, who has been in touch for many years past with this side of his activities, would place on record a tribute to the memory of one who was a great physicist and a great engineer. Sir Alfred was, indeed, throughout his long working life as much physicist as engineer. His interest in the purely physical side of thermodynamics was as great as his interest in its applications. He possessed a deep knowledge of the work of the founders of the science, and Sadi Carnot's immortal *Réflexions sur la Puissance Motrice du Feu* drew from him unstinted admiration. His own contributions to the science, his *Thermodynamics for Engineers* and his articles in the *Dictionary of Applied Physics* have moulded the thought of some generations of students, and their influence is still profound.

In 1897 he established his reputation as a leader amongst writers on refrigeration by a course of Howard lectures delivered to the Royal Society of Arts. These lectures were published eleven years later under the title *The Mechanical Production of Cold.* The book, charming and lucid in its style, is a classic in the literature of science.

Sir Alfred was the Chairman of a Committee of the Institution of Mechanical Engineers which was set up "to consider and define a standard in refrigeration and its units of measurement". This committee published a report in 1914 in which was given a series of tables and charts of the thermodynamic properties of refrigerants.

When the Food Investigation Board was established, Sir Alfred accepted the chairmanship of its Engineering Committee and continued, without missing a meeting, to preside until his death. He made an admirable chairman, and an investigator of nervous temperament, when presenting his report, had the path smoothed for him by Sir Alfred's genial manner and his clear exposition of the salient points. It was an invariable rule with him to go through the report with its author before the meeting, and any obscurity was cleared up before presentation.

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* *Times*, January 8; *Engineering*, January 11; *Nature*, January 26, 1935. Sir Alfred's autobiographical volume *An Engineer's Outlook* (Methuen) should also be consulted.

He was keenly interested in the work of the International Union of Physics and took an active part in the work relating to thermodynamic symbols and the definition of the thermal unit. One of his last public appearances was at the meeting of the Union in London in October 1934, when the report of the Symbols, Units and Nomenclature Commission came up for discussion.

His work as Principal of a large university left him, in his later years, small opportunity for research, but on one occasion—when a flowmeter was under consideration—Sir Alfred's enthusiasm was roused to the point of commandeering his bathroom for experiments. An account of the work was published in the *Proceedings of the Royal Society of Edinburgh*. It is interesting to note that this type of flowmeter—a ball in a conical tube—had been suggested by him 50 years previously.

He was a Fellow of the Physical Society for 37 years, and served a term of office as Vice-President.

Sir Alfred Ewing was loved by all who came into intimate contact with him: in losing him the world of science has lost an outstanding personality.

EZER GRIFFITHS

DAVID KING MORRIS

Dr David King Morris, a Life Fellow of the Physical Society, passed away after protracted ill-health on May 14, 1935, at the age of 61 years.

Dr Morris became a student in the Engineering Department of University College, London in 1891 under Profs. Hudson Beare, Carey Foster, and J. A. Fleming, and was able to maintain himself at college for the full four-year period by means of the prizes, scholarships and exhibitions which he gained. In the third and fourth years he carried out in conjunction with the present writer research work on dielectric hysteresis and on the measurement of currents in inductive circuits; these were published by the Royal Society and in the *Philosophical Magazine*. He then proceeded as an 1851 Exhibition Scholar to the University and Polytechnicum of Zürich, where he was awarded the degree of Ph.D. in 1897 for a dissertation entitled "The Magnetic Properties and Electrical Resistance of Iron as dependent on Temperature", and on his return to England he was elected a Fellow of University College at the early age of twenty-four.

He was successively demonstrator at the S.W. Polytechnic (London) and Lecturer in Electrical Engineering at the Mason University College (which later became the University) at Birmingham. In the latter position Dr Morris was responsible for the entire work of the Electrical Engineering Department and in addition to his teaching duties much valuable research work was carried out or inspired by him.

The equipment of the new laboratories, admittedly at the time one of the finest in the country, was undoubtedly the result of his experience and foresight.

In the years 1904–1908 he contributed four joint papers to the Institution of Electrical Engineers. The paper on eddy-current brakes for testing motors (1905) was in front of its time, and the mathematical theory developed by him is now found to be of real service in engineering practice in the design of brakes for electricity-meters.

He later helped to found the firm of Morris and Lister at Coventry.

During the war as director of the M.L. Magneto Syndicate he became one of the chief suppliers of magnetos for aircraft.

A. W. P.

ARTHUR LYNCH

Colonel Arthur Lynch, who was born at Smythesdale, near Ballarat, in Australia in 1861 and died in a London nursing home this year, was almost a legendary figure. To some who were permitted to glimpse his achievements in the most diversified fields he may well have appeared something like a universal genius, and that is an interpretation of his character which he would not have put himself to any particular pains to contradict. Others, meeting him in his more expansive moods, may have been inclined to murmur "charlatan", in which they would have done him less than justice.

To the public in the opening years of the century Colonel Lynch was "the man who fought for the Boers" and was sentenced to death and reprieved. Opinions differ about the importance of the Irish Brigade which he is understood to have commanded, but it may be said with some confidence that it made no great difference to the military history of the campaign, and it was a wise act of clemency that made it possible for the rebel "Col." Lynch of the Boer war to be colonel of the 10th Battalion, Munster Fusiliers in France in 1918. From 1901 to 1903 he sat in the House of Commons as Nationalist member for Galway and he returned to the House in 1909 and sat until 1918 for West Clare. When the propriety of putting him forward in 1909 was challenged by a timorous member of the Selection Committee on the ground that he had "killed Englishmen", another member with an acid wit which Lynch himself could appreciate, retorted that the Colonel had killed more Englishmen when he was a doctor in London than when he was a soldier in South Africa.

That was not a verdict he would have accepted, for he took himself seriously as a scientist. "I have had", he used to claim, "a longer academic training than any other man living." After leaving Melbourne University with degrees in arts and engineering, he studied at Berlin University, took the diploma of the École Supérieure d'Électricité in Paris, followed a medical course at the Hôpital Beaujon in Paris and qualified for the Medical Register in this country by taking the conjoint examination from St Mary's Hospital, entitling him to inscribe himself M.R.C.S., L.R.C.P.

His engineering studies were interrupted by the attractions of journalism in Paris, and it is probably a sound estimate of his career to say that he was essentially a journalist. He had that convenient type of mind that enables its possessor to pass any examination with little effort, and a real flair for "news value". To this must be added an Irishman's love of being against the conventions. He was a warm supporter of Spahlinger's claims in the field of tuberculosis, but a caustic and unsparing critic of the psycho-analysts.

His attitude towards scientific questions is embodied in a pungent volume which he wished to call *Humbugs of Science*. The late Sir John Murray, however, preferred the more decorous title *Science, Leading and Misleading*, under which it ultimately appeared. But the fight which he considered "the best and the last" was against Einstein. He regarded relativity physics as a confused sort of metaphysics and ultimately got his book published. His confident declarations to publishers that he could "knock Einstein out in one round" delayed the issue.

Lynch published novels, poems, a two-volume book on psychology which, he claimed, laid the foundations of a new and true system, and a book on the future of the Empire. The book he most wanted to write at the end was a defence of republicanism, in which he held a passionate belief that not even first-hand experience of republics had succeeded in eradicating.

REVIEWS

Electron Emission and Adsorption Phenomena, by J. H. de Boer. Translated from MS. by Mrs H. E. Teves-Acly. (Cambridge: The University Press.) 21*s*.

The author of this book is on the research staff of the Philips Lamp Factory at Eindhoven. It contains an account of thermionic and photoelectric effects, both external and internal, written with special reference to the influence of adsorption at the outer surfaces and internal flaws of the material. Following a preliminary account of thermionic emission, the second chapter gives a careful discussion of the forces which can give rise to adsorption, and their relative importance in the case of atoms and ions, polar and non-polar systems. Apart from a digression in chapter vii to give an account of the photo-ionization of gaseous molecules and the Franck-Condon principle, the remainder of the book shows how the ideas of the second chapter can be applied first to the emission of electrons from complex surfaces such as are widely used for dull-emitter filaments, and next to the phenomena of internal conduction associated with the names of Gudden and Pohl.

A very large number of experimental results are described, including many of the author's own; these, however, seem to have been selected with a view to illustrating the theory rather than to giving a complete account of the subject. For example, in the chapter on the selective photoelectric effect only two lines deal with the liquid sodium-potassium alloy with which so much of the work has been done. Again, more reference might have been made to the work of Przibram and his school on the coloration of alkali halides.

The author's views, which are a development of ideas due to Langmuir, Becker and others, are well set out. On a basis of the forces between surfaces and the atoms adsorbed on them he is able to explain a wide range of phenomena, many of them of commercial importance. Objections are dealt with fairly, and the general impression with which one is left is that the author has succeeded in presenting a probable explanation of the main facts and one capable of extension in other directions.

The style is, in some places, decidedly clumsy, perhaps as a result of too literal a translation, but in spite of this the book is readable. G. P. T.

The Optical Basis of the Theory of Valency, by R. de L. Kronig, Ph.D. Pp. x+246. The Cambridge Series of Physical Chemistry. (Cambridge: The University Press, 1935.) 16*s*. net.

Knowledge of the nature of chemical valency has developed rapidly in recent years from G. N. Lewis's hypothesis of octets and sharing of paired electrons to the more detailed theories of Heitler and London (1927), Pauling and Slater (1931) and Hund and Mulliken (1932–3). This development is a consequence of the progress made in the quantum-theoretical interpretation of molecular spectra, and has been achieved very largely by the application of purely physical methods. It is not only the chemist who has found difficulty in keeping abreast of this outgrowth of modern physics and felt the need of a clear exposition in English book form. To present the subject adequately within the rather narrow limits of some 250 pages is no easy task for any author, be he theoretical physicist, physical chemist or spectroscopist. However, it has been achieved most admirably in this work by Dr Kronig, who has already won world-wide fame not only by his important contributions to modern theoretical physics but also by his powers of lucid exposition of the essentials of the subject in our language as well as his own.

Taking advantage of the fact that detailed accounts of the analysis of atomic and molecular spectra and of the quantum-mechanical theory of spectra and valency are available in other recent books, Dr Kronig has reduced to a minimum both the mathematical argument and the spectroscopic detail, thereby widening the scope of the work and making it more acceptable to the chemical reader. As will be seen from the following outline it is not entirely spectroscopic, nor is it for the chemist only. The opening chapter deals with relevant chemical facts, atomic theory, atomic models, etc., the next with the investigations of atomic and molecular structure by means of X-ray and electron diffraction. The third chapter is given to atomic spectra and the periodic system, and the next two to band spectra and chemical binding in diatomic and in polyatomic molecules. The phenomena of optical and thermal dissociation are dealt with in the last chapter. A table of atomic electron configurations and ground states is appended.

Some of the notes made in the course of a first reading may not be out of place here. (i) It is clear from the first paragraph of p. 88 that α in equation (21·15) is a positive quantity; hence equations (21·16) and (21·20) must, if they are to agree with observation, have + in place of their − signs, whatever the theoretical justification for the latter may be. (ii) It is no longer true of all the measurements of rotation bands that, as is stated on p. 95, they were made in absorption and so refer to $v=0$. Actually Strong has measured fairly extensive rotation bands of HCl in emission for $v=0$ (to $J=33$) and for $v=1$ (to $J=23$). (iii) The table of constants for the ground states of diatomic molecules on pp. 100–4, in which the author has supplemented the data in this Society's report by more recent data (to April 1935), is by no means complete; many emitters recorded before that date are omitted, and for a few others the data are not the latest. NiH looks ill at ease in the class of halogen hydrides on p. 101. (iv) The proximity of the dissociation energies of N_2 and CO is closer than appears on p. 118, the latest value of the latter being as low as 8·41 V. as against the earlier estimate of 10 V. quoted there and on p. 136. (v) The spectrograms in the plates facing pp. 170 and 222 would gain in usefulness if provided with scales of either wave-lengths or wave-numbers. (vi) In the 12th line from the foot of p. 236 either the two ω_e values should be interchanged or the word "respectively" should be deleted.

This is an important work, which should find an even wider circle of readers than the author's well-known *Band Spectra and Molecular Structure*. As usual, the work of the Cambridge University Press is beyond praise.

W. J.

La Spectroscopie appliquée, by P. SWINGS. Pp. 188. (Bibliothèque scientifique Belge. Paris: Hermann et Cie, 1935.) 15 francs.

In spite of the great progress made in spectrographic analysis in recent years, it appears that the method has not been applied in Belgium to nearly the same extent as it has in many other European countries and the U.S.A. To remedy in some measure this unsatisfactory state of things is the main object of this excellent little monograph, which is based to a large extent on a course given by the author in the Technical Faculty of the University of Liège, and is the first of its kind in the French language. An entirely practical point of view is adopted throughout. After an introductory chapter on the history, advantages and physical basis of the spectroscopic method of analysis, we have one chapter on instruments (spectrographs and light-sources), two chapters on qualitative and quantitative analysis by means of emission spectra, and two on typical applications to chemical, technical and biological problems. Then follow a chapter on analysis by means of absorption spectra, one on other methods (infra-red, X-rays and fluorescence), and one outlining the physical interpretation of atomic and molecular spectra. A useful bibliography and a folding plate of spectrograms are appended. This small book contains a remarkable amount of useful information, which is both well-selected and well arranged. It can be thoroughly recommended to English readers. W. J.

Molekülspektren und ihre Anwendung auf Chemische Probleme: I, Tabellen, by H. SPONER. Pp. vi+154. (Berlin: Julius Springer, 1935.) RM. 16, bound RM. 17.60.

The usual plan of tabulating numerical data in an appendix to the main text of a comprehensive work, especially one on such a wide and rapidly growing subject as molecular spectroscopy, has the disadvantage that it makes no provision for two probable requirements: many readers may require the tables only, and the tables may require revision and expansion more frequently than the text. Prof. Sponer and her publishers have therefore done wisely in issuing these valuable tables as a separate volume, which is to be followed by the text in volume II. The work is an addition to the well-known series of *Struktur und Eigenschaften der Materie* monographs founded by Professors Born and Franck.

Both diatomic and polyatomic molecules are included, each class occupying about a half of the volume. For diatomic molecules there are tables of constants for all the known electronic states and band systems, observed predissociation effects, isotope effects, alternating intensities and nuclear spins, and electron-impact processes. Most of these are founded, as Dr Sponer very handsomely acknowledges, upon the corresponding tables (Appendix II and Tables 12 and 26) of this Society's Report on the subject, supplemented where possible by data as to the electron configurations and dissociation products of the various molecular electronic states, as well as by new data from original papers that have appeared since 1932. Those chemists and spectroscopists who have pointed out the need of a new edition of Appendix II will certainly find here all they require for the time being. The tables for polyatomic molecules are entirely new; they cover infra-red absorption and Raman spectra (with diagrams of the modes of vibration of the various molecules in their ground states), electronic bands, and electron-impact processes. The work is brought up to date as far as possible by two supplementary tables containing matter which was published in 1934 and early 1935 while the rest of the volume was going through the press. Each group of tables is preceded by a few explanatory paragraphs and followed by a list of references to original papers, and an adequate eight-page index is provided.

Great care has obviously been taken in the preparation of the tables, and the omissions and numerical slips so far detected are very few indeed. Perhaps it would be well to point out here that the entries 6·266 and 5·181 on p. 13 for two states of Li_2 are the values of $2x_e\omega_e$, not $x_e\omega_e$ itself; this error is the result of a rather unusual method of presentation in the original paper. Spectroscopists and physical chemists will undoubtedly find these tables invaluable and will look forward to the appearance of the text itself. W. J.

Optical Rotatory Power, by T. M. LOWRY, C.B.E., F.R.S. Pp. xiii+483. (London, Longmans, Green and Co., Ltd.) 30*s*. net.

This is, we believe, the first original English work on a subject which, during the century or more that has passed since Biot's discovery, has attracted much attention, and has accumulated a wealth of special literature. Prof. Lowry's volume is hardly likely to find a rival in our generation. His treatment of the subject which he has made so peculiarly his own is scholarly, full and well-balanced, and the citations from the literature are detailed enough to save even the specialist much rummaging in the dusty files of the original papers.

The scope of the book may be indicated sufficiently by the titles of the four parts into which it is divided—*Historical and General*, *Polarimetry*, *Special Cases*, and *Theoretical Considerations*. Theory and experiment are held admirably in balance, the book is well up to date—Born's latest views receive adequate and detailed treatment—and the descriptions of apparatus and instruments leave nothing to be desired. The book, indispensable to the physical chemist, may be read with pleasure and profit by anyone interested in the history and advance of purely physical science. A. F.

The Theory of Atomic Spectra, by E. U. CONDON, Ph.D., and G. H. SHORTLEY, Ph.D. Pp. xiv+441. (Cambridge: The University Press, 1935.) 42*s*. net.

The theoretical interpretation of the structure of line spectra of neutral and ionized atoms has now extended over a little more than two decades. In the first Bohr's quantum theory, a combination of quantum postulates with classical kinematics and dynamics, was developed, and its limitations were ascertained. In the second the new dynamical theory was developed along two different lines, the matrix mechanics by Heisenberg and the wave mechanics by Schrödinger, afterwards unified by Dirac to form the system of quantum mechanics, in terms of which all the observed structural features of atomic spectra are much more completely understood and at least semi-quantitatively explained. In addition to the many books of various degrees of difficulty which have appeared on quantum mechanics itself, we have had several on the theoretical interpretation of atomic spectra, which, with one notable exception, were written from the point of view of Bohr's correspondence principle. The exception is White's *Introduction to Atomic Spectra* (McGraw-Hill, 1934) which meets the requirements of those taking up the subject for study in the laboratory, especially if it is used in conjunction with Bacher and Goudsmit's *Atomic Energy States* (McGraw-Hill, 1932). Although several fundamental theoretical problems still await solution, and more detailed theoretical calculations on atomic spectra are desirable, the present state of the interpretation of such spectra may be regarded as fairly closed and highly satisfactory, and, therefore, a fitting subject for a complete and authoritative survey such as is provided in this masterly treatise.

In the introductory chapter the authors give a brief and well-balanced historical sketch, and in the second an exposition, along the lines laid down by Dirac, of the quantum-mechanical principles used in the rest of the work. The next two chapters are devoted to the presentation of special results of the general theory which are used throughout the book. The detailed development of the theory of atomic spectra begins with chapter v and continues as a unified and logical deduction of their structure from quantum mechanical principles throughout the remaining fourteen chapters. The theory is given in complete detail and the results are compared with experimental facts, but no attempt is made to adhere to chronological order. While admitting the value of the group theory in atomic physics the authors derive all the results without it in order to avoid unnecessary addition to the mathematical apparatus; there are, indeed, already three recent books by Weyl, Wigner and van der Waerden in which atomic spectral theory is regarded from the standpoint of the group theory. A discussion of universal constants and natural atomic units is appended. The book is well indexed, and, as goes without saying, magnificently produced. To describe it as a monumental work is no idle repetition of a reviewer's stock phrase; it really is one. It will be especially welcome to those spectroscopists who have for some time past had to seek theoretical results in widely scattered papers, as well as by those who wish to continue their reading beyond White's book, which, indeed, will serve as an excellent introduction to this fuller work. W. J.

A Text-book of Light, by L. R. MIDDLETON. Pp. viii+288. (London: G. Bell and Sons, Ltd.) 6*s.*

This book covers the ground required for the subject of Light in the Higher School Certificate and the Intermediate Science examinations, although it aims at a treatment rather more general than that of a text intended only for preparation for such tests.

It may be said at once that the author has a clear style of exposition. The general scope of the book is well conceived, and an attempt is made to awaken interest in optical instruments and the phenomena of physical optics. It is disappointing, however, to find that he has completely ignored the recommendations of the Physical Society's *Report on the Teaching of Geometrical Optics*, so that the use of the book can only tend to perpetuate the unhappy confusion and division already existent between school optics on the one hand and technical optics on the other. Furthermore, the most objectionable features of the usual school optics, such as the use of unsymmetrical equations of the type

$$\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}$$

are again encountered.

While a desire to awaken an interest in optical instruments is praiseworthy in an author, the appearance of such general statements regarding a telescope as

$$\text{``magnifying power} = \frac{\text{diameter of objective}}{\text{diameter of eyepiece}}\text{''}$$

(p. 123) may very justly disturb our confidence in the author's wisdom in venturing off the beaten track. The paragraph on "night glasses" is really entirely concerned with explaining the enhanced visibility of stars obtained with a telescope, and has little or nothing to do with "night glasses" in the ordinary sense of the term. The whole section is most misleading, and the same thing must be said of some other parts of the book.

Much better things than this must be expected before there can be any valid reason for abandoning some of the older school books of the same type. L. C. M.

Origins and Development of Applied Chemistry, by Prof. J. R. PARTINGTON, M.B.E., D.Sc. Pp. xii+597. (London: Longmans, Green and Co., 1935.) 45*s.* net.

In spite of all that has been written on the subject during the last thousand years, the plain truth is that the origin of chemistry remains an unsolved mystery. The course of the science from Islam to Latin Christianity, and its subsequent growth to the majestic proportions that characterize it to-day, are clear enough. It is also pretty well established that the Muslims obtained their initiation into chemical knowledge mainly by way of Alexandria, and that the Alexandrian Greeks derived much from Egyptian sources. But the simple hypothesis that chemistry arose *ab initio* in ancient Egypt, and that the chemical succession is unbroken through Moses, Mary the Copt, Zosimos the Panopolitan, Khalid, Jabir, Razi, Roger Bacon, Albert the Great, Paracelsus, Lémery and Boyle, has little but its simplicity to recommend it. We now know, for example, that Assyria possessed competent practical chemists as early as the seventh and sixth centuries B.C.; that alchemy in China is at least as ancient as in Alexandria, and possibly much older; that much the same is true of chemical theory and practice in India; and that Persian alchemy shows many signs of a very remote indigenousness. The wealth of discoveries of this kind published during the present century has thus not made the problem of the origin of chemistry any easier; on the contrary, it has shown that the whole matter is far more complicated than had been suspected, and that a great deal of research will have to be carried out before any sort of finality is reached. It has also become clear that the data for speculation are insufficient, and consequently require to be supplemented by the patient accumulation of as many relevant facts as possible.

This kind of work is exacting, tedious and pedestrian. It offers no alluring prospect of spectacular discoveries, and demands the closest attention and perseverance in the establishment of detail. It is, in short, the kind of work that everybody would like to see done, but hopes that someone else will do. Prof. Partington has unselfishly attacked the task, and all who read the present book will agree that he has made a noble beginning. He is, of course, admirably equipped for the purpose, most especially in his unswerving determination to be accurate, to give line and word of his authorities, and to refrain from supposition; with Newton he would say *hypotheses non fingo*. He has, however, the additional qualification of possessing as wide a knowledge of old chemical literature as any man now living, with the probable exception of Prof. E. O. von Lippmann—and von Lippmann read the greater part of the book while it was still in manuscript. There could thus have been no more appropriate a choice for the work, and we must count ourselves fortunate that Prof. Partington, rejecting fields in the history of chemistry where honour is more easily to be gained, has methodically dug ground too intractable and too arid for any of less resolution than he.

The plan of the book is simple. Prof. Partington deals in turn with Egypt, Babylonia and Assyria, the Aegean Civilization, Troy and Cyprus, Asia Minor, Persia, Phoenicia, and Palestine. In each part, a short account of the general history and archaeology of the region concerned is given first (an extremely useful feature which the reader will appreciate) and this is followed by sections on metals and non-metals; each section is further divided into sub-sections, within which individual metals, minerals and other substances are separately considered. By this arrangement it is easy for the reader either to get a view of the technical chemical knowledge of a particular region, or to turn up all that was known of a given substance in the whole of the various regions.

It is difficult in a review to indicate the wealth of facts which Prof. Partington has amassed, docketed, arranged and fully indexed. There are, for instance, over 100 index references to lead and about the same to iron, while gold, as might be expected, occupies more than half a column of the subject index. Within the limits he has chosen, it would be hard to find any fact of importance that Prof. Partington has omitted; but not content with mere compilation, he has added comments and corrections from the chemist's point of view in cases where the facts reported are obviously doubtful or erroneous.

The general impression left by the book is one of surprise at the wide, deep and accurate knowledge of an empirical chemical kind possessed by the nations of antiquity. It seems, also, as if no one nation showed any remarkable pre-eminence over the others, with the possible exception of Egypt; and it is thus not very hazardous to suggest that research will have to be pushed back still further before the period and region of the principal discoveries—e.g. the smelting of ores—can be definitely fixed. Furthermore, the problem of the origin of the corpus of alchemical doctrine does not yet make contact with the chemical facts recorded by Prof. Partington; but his book is a long step towards the provision of a *terminus a quo* in this connexion.

Those who know the discipline of historical research in science will best appreciate Prof. Partington's achievement; but it would be a mistake to look upon his book as one for the specialist only. The general reader interested in the story of man's gradual conquest of the material world will find it a storehouse of relevant and reliable information, presented in a scholarly and attractive manner.

E. J. HOLMYARD.

A Symposium on Illumination, edited by WEBBER GRIEVESON, B.Sc., M.A., with a foreword by Lt.-Col. K. EDGCUMBE. Pp. xv + 229. (London: Chapman and Hall, Ltd.) 13*s*. 6*d*. net.

In 1933 a series of lectures was given (mainly at the Regent Street Polytechnic) under the auspices of the National Illumination Committee and the Illuminating Engineering

Society. The lectures, which are reprinted in this volume, demonstrate admirably the amazingly rapid advances made in this, as in so many other branches of physics, pure and applied. The topics considered are best shown by a catalogue of the lecture titles: Lighting in the Service of Mankind (C. C. Paterson); Radiation—Light and the Eye (J. W. T. Walsh); Electric Lamps and their Characteristics (W. J. Jones); Gas Lamps and their Characteristics (F. C. Smith); Daylight: Coloured Light (J. W. T. Walsh); Photometric Standards, Lamp Photometry and Illumination Measurement (J. T. MacGregor-Morris); The Redistribution of Light (G. H. Wilson); Lighting for Decoration and Entertainment (A. B. Read); Public Lighting (J. F. Colquhoun); and Lighting for Safety, Health and Welfare (H. C. Weston).

The treatment is what has been called semi-technical; it is popular in the best sense of the word; it is never cheap; and the lectures hold the reader's attention from the opening sentence of Mr Paterson's brilliant discourse, to that last page on which Mr Weston gives sage advice on spectacles for fine work.

A book to be thoroughly commended.

The Scientific Journal of the Royal College of Science. Vol. v. Pp. 137. (London: Edward Arnold and Co.) 7*s*. 6*d*. net.

The topics dealt with in the fifth volume of this journal are perhaps a little more pedestrian than those discussed in previous issues. Nevertheless the volume maintains the high standard of its predecessors, and contains adequate reports of some very valuable lectures. Seven addresses are reported under the heading of the Chemical Society's section, four under the Natural History section, and three only under the section allotted to the Mathematical and Physical Society, of which one address of special interest—that of Prof. Milne on the Expanding Universe—is reported only in abstract. We trust that succeeding issues of this journal, the importance of which increases with each year of issue, will see a more even distribution of topics between the sections allotted to the three Societies concerned. A. F.

Actualités Scientifiques et Industrielles. (Hermann et Cie., 6 Rue de la Sorbonne, 6, Paris.)

Under the above general heading we have received the monographs listed below. Each is written by an authority on his subject and the treatment is, in general, concise and clear.

222. Y. Rocard. *Propagation et Absorption du Son.* 15 fr.
225. H. Mineur. *Dénombrements d'Étoiles.* 15 fr.
230. P. M. S. Blackett. *La Radiation Cosmique*, I. *Aperçu Général.* 10 fr.
231. P. M. S. Blackett. *La Radiation Cosmique*, II. *La Méthode de la Chambre de C. T. R. Wilson.* 8 fr.
232. P. M. S. Blackett. *La Radiation Cosmique*, III. *L'Action du Champ Magnétique Terrestre.* 7 fr.
233. P. M. S. Blackett. *La Radiation Cosmique*, IV. *La Perte d'Energie par Ionisation.* 10 fr.
234. Y. Rocard. *La Stabilité de Route des Locomotives.* 15 fr.
236. Pierre Fleury. *Généralités sur les Mesures.* 15 fr.
241. P. Swings. *Les Spectres des Nébuleuses Gazeuses.* 10 fr.
245. Manuel Valadares. *Transmutation des Eléménts.* 10 fr.
247. G. Guében. *Structure Nucléaire.* 10 fr.
248. B. Rossi. *Rayons Cosmiques.* 12 fr.

X-Rays in Theory and Experiment, by A. H. COMPTON and S. K. ALLISON. Pp. xiv+828. (London: Macmillan and Co., 1935.) Price 32*s.* 6*d.*

Compton's *X-Rays and Electrons*, published in 1926, was a book of very strongly marked individuality; it was, moreover, largely given up to the study of a relatively small number of the outstanding X-ray problems of its time. This being so, it is more than usually interesting to see how the book transforms, nine years later, into a second edition.

In the first place, there has been an inevitable increase in bulk; the new work is more than twice the size of its predecessor. Secondly, Compton and Allison have transformed the book into a systematic and just sufficiently encyclopaedic treatise which covers a considerable portion of the whole domain of the physics of X-rays. In brief, the change in the book is well enough indicated by the change in the title; the new edition justifies the new title, comprehensive though that is.

The chapter headings give the clearest indication of the scope of the work. They are: (1) Discovery and Properties of X-rays (55 pages); (2) Production of X-rays (60 pp.); (3) Scattering of X-rays (147 pp.); (4) Dispersion Theory applied to X-rays (53 pp.); (5) Study of Crystal Structure by means of X-rays (49 pp.); (6) Intensity of Reflection of X-rays from Crystals (107 pp.); (7) Phenomena associated with the Ejection of Photo-electrons by X-rays (111 pp.); (8) Interpretation of X-ray Spectra (89 pp.); (9) Some accurate Methods of X-ray Wave-length Measurement and their Results (85 pp.). There are eleven Appendices of very varied content, which includes some valuable absorption data, as well as matter of more theoretical interest.

In general most stress is laid upon those divisions of X-ray research which are most germane to the major problems of atomic structure and of radiation. Other sections—on which one would normally consult such treatises as those of the Braggs or Siegbahn—are developed as a rule in less detail. This is not, however, to say that they are inadequately handled: in fact their salient features are in general depicted in a most illuminating fashion. The result is that while the book is still very directly addressed to the X-ray specialist, it is also fitted to serve as a scholarly and inspiring introduction to the subject for readers whose interests are less localized. There is an extensive bibliography for which completeness is not claimed, but which is ample for the needs of those beginning serious research in the physics of X-rays.

It is pleasant to note that in the course of its revision and expansion the book has not lost its original air of spontaneity; it still smells healthily of the laboratory. In other words the authors, having played an active part in building up the experimental material which they discuss, have naturally succeeded in presenting that material as part of a live and vigorous organism.

This is a notable and much-needed addition to the literature of X-rays, and we are fortunate in more ways than one in having it presented to us in English. It is salutary to think of the price which would have been demanded for a treatise of similar extent if it had happened to bear the title "Theoretisches und Experimentelles über Röntgenstrahlen". As it is, we have a great deal of good book for a relatively very modest price.

Printing and production are excellent, but it would assist cross-reference (especially in view of the way in which the equations are numbered) if the chapter numbers were inserted at the head of each page. For this, the present reviewer would willingly sacrifice the subject titles which head alternate pages, and which must have been far more troublesome to draft and print.

H. R. R.

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